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AN INVESTIGATION OF THE LIQUID LEVEL AT THE WALL OF A SPINNING TANK

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ABSTRACT

Results are presented for an analytical and experimental study of the liquid-level rise in a spinning cylindrical tank. Theoretical expressions for the steady-state free-surface contour and the transient liquid-level rise were derived. The experiments were conducted in a cylindrical Plexiglas tank operating at tangential velocities from 51.36 to 138.7 in./sec for viscosities ranging from 1 to 260 centistokes. The effects of initial liquid level, viscosity, and tangential velocity on liquid-level rise were determined; the results were compared with theory. An empirical correlation was also found for all the test conditions.

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TECHNICAL NOTE D-1536

AN INVESTIGATION OF THE LIQUID LEVEL AT THE WALL OF A SPINNING TANK

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SUMMARY

An analytical and experimental investigation was conducted to obtain information on the steady-state and transient liquid-level rise in a spinning cylindrical tank. Analytical expressions for the steady-state free-surface contour for both unexposed- and exposed-bottom conditions were developed. A theoretical relation based on simplifying assumptions was derived for transient liquid-level rise.

Tests were conducted in a cylindrical Plexiglas tank with a diameter of 26.5 inches. Various water-glycerine mixtures with viscosities ranging from 1 to 260 centistokes were used. The tank was operated at tangential velocities from 51.36 to 138.7 inches per second and with initial liquid levels ranging from 4 to 16 inches.

Experimental results showed that the rate of rise of liquid level at the wall increased with increasing fluid viscosity, decreasing initial liquid level, and increasing tank tangential velocity. The last of these tended to decrease as the initial liquid level decreased.

Although good agreement between experiment and theory was observed for the steady-state conditions, poor agreement was obtained for the rate of level rise for the transient case. The disagreement was attributed to the fact that radial- and axial-flow terms were neglected in the development of the transient solution. A correlating equation was found that allows for the prediction of transient liquid height at the tank wall for the conditions investigated.

INTRODUCTION

Solid-propellant rockets have been successfully stabilized during flight by being spun about their longitudinal axes. Use of this method of stabilization for liquid-propellant rockets is complicated by the induced motion of the liquid in the propellant tanks. An important aspect of the liquid motion is the rise of the liquid surface at the wall of the spinning tank. As the liquid rises, it can fill vents and inlets and may thereby spill or obstruct flow; furthermore, the location and amount of wetted tank surfaces influence the rate of propellant evaporation due to aerodynamic heating. Because of these considerations, it is important to be able to predict the height of the liquid at the tank wall at any time.

An analytical solution for the steady-state unexposed tank bottom is given in references 1 and 2. A theoretical solution for the transient variation of fluid angular velocity in a spinning tank was found (ref. 3), but the simplifying assumptions involved in the solution made its use questionable for the prediction of liquid-level rise. An analytical and experimental program was, therefore, conducted to study this problem.

Analytical solutions of the surface contour for the steady-state spinning condition for both exposed and unexposed tank bottom were developed. An attempt was made to find a solution for the liquid-level rise at the tank wall during the transient condition; the simplified mathematical model of reference 3 was used.

An experimental program was initiated to check the theoretical results and to study the influence of tank tangential velocity, initial liquid depth, and viscosity on the liquid-height - time relation at the wall of a right cylindrical Plexiglas tank. The tank had a diameter of 26.5 inches and was run at four tangential velocities ranging from 51.36 to 138.7 inches per second with five liquid viscosities ranging from 1 to 260 centistokes. Nearly all the experiments were performed at four initial liquid levels. Finally an empirical relation for the rise in liquid level at the tank wall was derived from the experimental results.

ANALYSIS

The problem was to find analytical solutions that would predict the transient and steady-state liquid-level rise at the wall of the tank. A right cylindrical tank partially filled with liquid was used in the investigation. If the tank is suddenly rotated at a constant angular velocity ω_w, the fluid will rise along the wall and form a concave free surface. When the fluid finally attains the angular velocity of the tank, a steady-state or equilibrium configuration will be reached.

For the general case of a fluid rotating in a vertical cylindrical tank, the forces acting at a point on the free surface of the liquid in the absence of surface tension are indicated in figure 1(a). Radial accelerations are caused by the fluid rotation $\omega^2 r$ and the radial components of the fluid displacements $\dot{q}_r$. Vertical accelerations arise from gravity g and vertical tank accelerations a and from the vertical component of the fluid displacements $\dot{q}_z$. Because of the displacement motions associated with the outflow, the vertical and radial accelerations will increase if fluid outflow occurs. Since the sum of all accelerations tangent to the free surface must be zero,

$$(\omega^2 r + \dot{q}_r)\cos\theta = (g + a - \dot{q}_z)\sin\theta$$

or

$$\frac{dr}{dz} = \frac{g + a - \dot{q}_z}{\omega^2 r + \dot{q}_r} \tag{1}$$

(All symbols are defined in the appendix.) The free-surface contour dr/dz can be obtained if equation (1), the force-balance equation, can be solved. Finding a solution to this equation requires a knowledge of the distribution of the radial and vertical accelerations and the fluid angular velocity.

Analysis of equation (1) was conducted for a nonaccelerating tank (i.e., $a = 0$) for steady-state and transient conditions as indicated in the following sections.

Steady-State Condition

When the fluid in the tank rotates at the same angular velocity as the tank and all displacement motions have ceased, a steady-state condition exists, and, as a result, ω becomes a constant, $\dot{q}_r = 0$, and $\dot{q}_z = 0$. Equation (1) can thus be integrated to yield

$$H = H_0 + \frac{\omega^2 r^2}{2g} \tag{2}$$

Equation (2) indicates that the free surface is a paraboloid of revolution.

That steady-state liquid height can be expressed in terms of the initial liquid level H_i is clear from a consideration of the steady-state volume given by

$$V = \pi R^2 H_0 + \frac{\pi R^2}{2}(H_w - H_0) \tag{3}$$

When equation (3) is equated to the initial liquid volume $\pi R^2 H_i$, the steady-state liquid height becomes

$$H = H_i + \frac{\omega^2 R^2}{2g}\left[\left(\frac{r}{R}\right)^2 - \frac{1}{2}\right] \qquad (4)$$

and the steady-state liquid height at the tank wall is

$$H_w = H_i + \frac{\omega^2 R^2}{4g} \qquad (5)$$

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Figure 1(b) shows a typical steady-state free-surface contour for a tank with an unexposed bottom.

It can be seen from equation (4) that for $r > \sqrt{R/2}$ the liquid level rises above the initial liquid level, while at $r = \sqrt{R/2}$ the steady-state liquid level coincides with the initial liquid level, and finally, at $r < \sqrt{R/2}$ the liquid level is depressed below the initial liquid level. It is also evident that $\Delta H_w = H_w - H_i = H_i - H_0$.

If $H_i < \omega^2 R^2/4g$, the bottom of the tank will be exposed because of the depression of the liquid as indicated in figure 1(c). From the consideration of initial and final liquid volume, it can be shown that

$$H^* = \omega R \sqrt{\frac{H_i}{g}} - \frac{\omega^2 R^2}{2g}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (6)$$

This equation describes a paraboloid of revolution (fig. 1(c)) between $r = R$ where $H_w^* = \omega R\sqrt{H_i/g} < H_w$ and $r^* = \left[R^2 - (2R/\omega)\sqrt{gH_i}\right]^{1/2}$ where $H^* = 0$. Figure 2 illustrates the influence of initial liquid level on ΔH_w^* as a function of tank tangential velocity. Exposing the tank bottom suppresses the liquid-level rise for a given value of tank tangential velocity.

Transient Condition

The angular velocity and acceleration distribution of the fluid as a function of space and time must be known so that the fluid-surface contour for the transient condition can be obtained. This requires the solution of the Navier-Stokes equations in conjunction with the continuity equation. For an incompressible fluid, these equations are:

$$\rho\left[\frac{Dq_r}{Dt} - \omega^2 r\right] = \overline{R} - \frac{\partial P}{\partial r} + \mu\left[\nabla^2 q_r - \frac{q_r}{r^2} - \frac{2}{r^2}\frac{\partial(r\omega)}{\partial\theta}\right] \tag{7}$$

$$\rho\left[\frac{D(r\omega)}{Dt} + \omega q_r\right] = \overline{\theta} - \frac{1}{r}\frac{\partial P}{\partial\theta} + \mu\left[\nabla^2(r\omega) + \frac{2}{r^2}\frac{\partial q_r}{\partial\theta} - \frac{\omega}{r}\right] \tag{8}$$

$$\rho\,\frac{Dq_z}{Dt} = \overline{Z} - \frac{\partial P}{\partial\theta} + \mu\nabla^2 q_z \tag{9}$$

$$\frac{q_r}{r} + \frac{\partial q_r}{\partial r} + \frac{1}{r}\frac{\partial(r\omega)}{\partial\theta} + \frac{\partial q_z}{\partial z} = 0 \tag{10}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + q_r\frac{\partial}{\partial r} + \omega\frac{\partial}{\partial\theta} + q_z\frac{\partial}{\partial z}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Boundary conditions for the problem being investigated are as follows:

(1) For no slip at the tank wall, $\omega = \omega_w$ at any time t.

(2) For the remainder of the fluid, $\omega = 0$ at $t = 0$ for $r < R$.

Solutions of the Navier-Stokes equations for the rotating-tank boundary conditions have not been obtained, however, because of the complexity of these equations. It was, therefore, necessary to consider simplifying assumptions in order to obtain a solution. A closed-form solution could be obtained if the problem were reduced to a one-dimensional transient case; this represents a rather severe restriction. Assuming that the fluid moves in concentric cylinders about the axis of rotation and that the velocity depends only on time and radius implies that $\overline{\theta} = q_r = 0$, $\overline{R} = -\rho\omega^2 r$, and

$$\frac{\partial^n X_i}{\partial u_i^n} = 0$$

where $X_i = q_r, q_\theta, q_z, P$ and $u_i = \theta, z$; and

$$\frac{\partial^n Y_i}{\partial v_i^n} = 0$$

where $Y_i = q_r, q_z, P$ and $v_i = r, t$. For these assumptions, equations (7) to (10) reduce to

$$\frac{1}{\nu}\frac{\partial\omega}{\partial t} = \frac{\partial^2\omega}{\partial r^2} + \frac{3}{r}\frac{\partial\omega}{\partial r} \tag{11}$$

Reference 3 presents the solution of this equation as

$$\frac{\omega}{\omega_w} = 1 - \frac{2R}{r}\sum_{n=1}^{\infty}\frac{J_1(\alpha_n r/R)}{\alpha_n J_2(\alpha_n)} e^{-\nu\alpha_n^2 t/R^2} \tag{12}$$

The increase in the height of the liquid at the tank wall is obtained when equation (12) is substituted into the force-balance equation (eq. (1)) for $\dot{q}_r = \dot{q}_z = 0$ and integrated over dr. The solution is

$$\frac{\Delta h_w}{\Delta H_w} = 1 - 16\sum_{n=1}^{\infty}\frac{e^{-\nu\alpha_n^2 t/R^2}}{\alpha_n^2} + 8\sum_{n=1}^{\infty}\frac{e^{-2\nu\alpha_n^2 t/R^2}}{\alpha_n^2} \tag{13}$$

where $\Delta H_w = H_w - H_i$ is the liquid rise at the wall for the steady-state condition (eq. (5)).

In view of the restrictive assumptions, the accuracy of equation (13) is questionable; however, it may be useful in indicating the influence of some of the important variables involved. For example, according to equation (13) the rate of rise of the liquid-level ratio at the tank wall should depend on the liquid kinematic viscosity and the radius of the tank.

APPARATUS AND PROCEDURE

A diagram of the test rig used in the experimental investigation is shown in figure 3(a). The test rig consisted of a transparent right cylindrical Plexiglas tank with an internal diameter of 26.5 inches and a height of 26 inches clamped between two flat plates and bolted to a motorized turntable. The turntable was rotated by a variable-speed motor capable of accelerating the liquid-filled tank to 138.7 inches per second in approximately 3 seconds.

Preliminary test runs revealed that a 1/8-inch misalinement of the tank's spinning axis with respect to the true vertical produced waves large enough to obscure the liquid height measurements. Careful shimming was required to obtain true alinement.

For each experiment the partially filled tank was rapidly accelerated from rest to a constant angular velocity while the motion of the liquid surface, the revolutions indicator, and the elapsed-time indicator were photographed until the free surface of the liquid appeared close to its equilibrium contour.

The fluid-surface contour was photographed in a darkened room by means of a shutterless, semiautomatic camera and a high-intensity light source; the light was synchronized to fire at a predetermined tank position. Instrumentation consisted of an elapsed-time indicator with an accuracy greater than 0.1 percent, an electrical tachometer with an accuracy of ±1 percent, and a metal scale, which was mounted on the wall of the tank, with an accuracy of ±0.05 inch. A typical photograph of the spinning tank and liquid motion is shown in figure 3(b). It was also possible from the photographs to establish the liquid-surface profiles if corrections were made for the distortion caused by the refraction of light from liquid to Plexiglas and from Plexiglas to air.

Experiments were conducted at speeds of 37, 60, 80, and 100 revolutions per minute, which correspond, respectively, to tangential velocities of 51.36, 83.28, 111, and 138.7 inches per second and liquid depths of 4, 8, 12, and 16 inches. Water and four water-glycerine mixtures were used to obtain kinematic viscosities of 1, 3.4, 34.5, 91, and 260 centistokes. Except for the 3.4-centistoke tests, runs were made at all liquid levels and tank speeds indicated. The combined conditions of maximum speed and maximum liquid depth caused the height of the liquid at the tank wall to reach the top of the tank before equilibrium conditions were established.

RESULTS AND DISCUSSION

Experimental Data

The results of the experimental runs indicated that, in the spinning-tank system investigated, the rate of rise of the liquid at the tank wall is a function of the variables investigated, namely, the tangential velocity of the tank, the initial liquid level, and the viscosity of the liquid. Table I is a compilation of the measured liquid levels at the tank wall and the corresponding times recorded during the course of the experimental program. Figure 4 shows a typical time variation of the rise in liquid level at the wall. The zero time shown in the plot and in the table was taken as one-half the time required for the tank to reach full speed. The maximum tank acceleration period was 3 seconds for the 138.7-inch-per-second runs.

Steady-state condition. - Theoretical values of the steady-state liquid-level rise ΔH_w calculated from equation (5) for the test conditions of initial liquid level and tangential velocity are shown in table II. For tests where the equilibrium condition was reached, the liquid height at the tank wall agreed with the theoretical values of table II within ±2 percent; this value corresponds to a known error of ±1 percent for the angular velocity of the tank. For convenience of analysis, the experimental liquid-level rise Δh_w is therefore presented as the ratio $\Delta h_w/\Delta H_w$ for each speed.

The surface contour for a typical steady-state condition is presented in figure 5(a). The experimental data fall within ±0.2 inch of the predicted contour.

Transient condition. - Plots of the surface profile at several intervals during the transient condition for a typical run are shown in figure 5(b). The axial symmetry in the surface profiles suggests that the motion of the liquid was truly axially symmetric.

Effect of viscosity. - Figure 6(a) is a representative curve that shows the general influence of viscosity on liquid-level-rise rate. In all runs, increasing the viscosity decreases the time required to reach any particular liquid height. This is to be expected because the shear forces operating at the higher viscosities are larger than those at lower viscosities.

Effect of initial liquid level. - A typical plot illustrating the general effect of the initial liquid level on the rate of rise at the wall for fixed viscosity and tank speed is shown in figure 6(b). The significant aspect of this plot is the shape of the curves. The liquid rises more rapidly as initial liquid level decreases because of the increasing influence of the tank bottom. As the initial liquid level and the corresponding liquid volume decrease, the tank wall communicates less torque to the liquid, while the tank bottom exerts a constant but relatively larger torque to the liquid.

More specifically, the rate of rise should be a function of the ratio of total liquid wetted surface (proportional to torque) to liquid volume (proportional to mass). For a cylindrical tank the aforementioned

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ratio is $2/R + 1/H_i$, which indicates that the rise rate should be a function of tank radius as well as initial liquid level. The diminishing effect on level-rise rate observed as initial liquid level is increased, is also indicated by this expression.

Effect of tank speed. - It was found that the variation of rate of rise of the liquid at the wall with tank tangential velocity was a function of initial liquid level. For the 16-inch-initial-liquid level (fig. 6(c)), the liquid-level rise increased with tank speed according to the relation $\Delta h_w/\Delta H_w = 1 - e^{-K\sqrt{\omega t}}$. The effect of tank speed diminished, however, as the initial liquid level decreased. The rise data for several viscosities for the 4-inch liquid level are shown in figure 6(d). Except for the run with a tangential velocity of 51.36 inches per second and a viscosity of 1 centistoke, the rate of rise was essentially independent of the tangential velocity of the tank for this initial liquid level.

Comparison of Experimental Results with Theory

Illustrative calculations of $\Delta h_w/\Delta H_w$ as a function of time for several viscosities and tank radii obtained from the theoretical relation of equation (13) are shown in figure 6(e). This figure shows that the rate of rise of the liquid increases as viscosity is increased and as tank radius is decreased. Figure 7 shows a comparison between the theoretical results of equation (13) for the test-tank radius and the experimental data for two viscosities. Equation (13) predicts a slower rate of rise of the liquid than that determined by the experiments. The agreement is poor for the two viscosities presented, but, as the viscosity increases, the agreement improves. The inadequacy of the one-dimensional approach is thus clearly indicated.

In view of the symmetric transient and steady-state surface profiles observed (fig. 5(b)) the assumption that fluid velocities and pressures are axially symmetric so that $\partial^n X_i/\partial\theta^n = 0$ is certainly valid. (This, of course, requires that the tank be symmetric with respect to its spinning axis and that the axis be parallel to the external force field.) Although the experimental data (fig. 6(b)) show that the rate of rise tends to decrease with increasing liquid level, it does not appear likely that a good correlation could be attained for the low viscosities even for very large initial liquid levels. It, therefore, seems reasonable that neglecting $\partial^n X_i/\partial z^n$ and the tank-bottom boundary condition in the theoretical approach produces only a small error relative to the total error. The primary causes of the analytical error are, therefore, probably the assumptions that $q_r = 0$ and $\partial^n Y_i/\partial V_i^n = 0$.

Empirical Correlation

Since the simplified analysis proved to be inadequate in predicting liquid-level - time relations, it became necessary to obtain an empirical correlation for the data variations. The data for all levels were found to fit an empirical equation given by

$$\frac{\Delta h_w}{\Delta H_w} = 1 - e^{-m \frac{\nu t}{R^2}} \tag{14}$$

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where the nondimensional parameter m in the exponent is a function of initial liquid height; m is plotted as a function of viscosity in figure 8. Equation (14) in conjunction with figure 8 allows for the prediction of liquid height for any time within the range of variables investigated. Figure 9 shows a fit of the equation to the experimental data for the 1-centistoke, 16-inch-liquid-level tests and the 260-centistoke, 4-inch-liquid-level tests. Figure 9(a) is indicative of the poorest agreement expected from the correlation.

Application of Results

Inasmuch as the program was conducted to provide some insight into the behavior of fluids in spinning propellant tanks of flight vehicles, it is of interest to discuss the possible applications of the results. Fluids of interest (e.g., liquid hydrogen, liquid oxygen, and hydrocarbon fuels) have kinematic viscosities lower than the range of values covered in the experiments. Extrapolation of the correlations of figure 8 is required for estimating the transient level rise of these fluids.

The effect of longitudinal tank accelerations on the steady-state condition can be found if $g + a$ is substituted for g in equation (2). Thus, the steady-state level rise becomes $\Delta H_w = \omega^2 R^2/4(a + g)$. For the transient level rise, the effect of a will be to change the steady-state height term ΔH_w, but the effect on the rise ratio $\Delta h_w/\Delta H_w$ is not clear. According to the formulation of equation (12), the solution for ω does not involve a. Thus, theoretically, the time required to reach any value of $\Delta h_w/\Delta H_w$ is not affected in equation (13). In view of the inadequacy of equation (13), however, no firm conclusions can be drawn about the effect of tank acceleration on the transient level rise.

Finally, fluid outflow might have some effect on the level rise characteristics because of the possible circulation motions set up by the discharge. An insight into such possible effects on the level rise could not, however, be obtained from a qualitative examination of the equations involved.

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SUMMARY OF RESULTS

The following principal results were obtained from an investigation of the fluid motion in a cylindrical tank spinning about its longitudinal axis:

1. Integration of the force-balance equation for the liquid free surface showed the steady-state liquid-level rise at the tank wall to be a function of the square of the tank tangential velocity. If the tank bottom becomes exposed because of small initial liquid-level heights, the steady-state level rise will be smaller than that for the unexposed case. The steady-state surface contour is a segment of a paraboloid of revolution for both exposed and unexposed conditions.

2. Experimental results verified the theoretically predicted level rise and surface contours for the steady-state condition. Transient contours were symmetrical about the axis of the tank.

3. Experimental results showed the rate of level rise at the wall increased with increasing fluid viscosity, decreasing initial liquid level, and increasing tangential velocity. The latter effect tended to decrease as the initial liquid level was decreased.

4. Poor agreement was observed between experimental time variations of level rise and theoretical values obtained from solutions based on the highly simplifying assumption of one-dimensional flow. The discrepancies became worse as viscosity decreased. The disagreements were believed to be due primarily to the neglecting of radial and axial flow terms and the boundary condition of the tank bottom in the theoretical solution.

5. Empirical relations were obtained to describe the experimental variations of liquid-level rise as a function of time, viscosity, tank tangential velocity, and initial liquid level.

Lewis Research Center
National Aeronautics and Space Administration
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APPENDIX - SYMBOLS

a	vertical acceleration of tank, in./sec^2
g	acceleration due to gravity, in./sec^2
H	steady-state liquid level, in.
ΔH	change of steady-state liquid level from initial level, in.
h	transient liquid level, in.
Δh	change of transient liquid level from initial level, in.
J_1	Bessel function of first kind, first order
J_2	Bessel function of first kind, second order
K	empirical exponent for curve fit, 1/sec$^{1/2}$
m	parameter in empirical exponent for curve fit
n	summation index, eqs. (12) and (13)
P	pressure, lb/sq in.
q_r,q_z,q_θ	polar velocities, in./sec
$\dot{q}_r,\dot{q}_z$	polar accelerations, in./sec^2
R	radius of tank, in.
$\overline{R},\overline{Z},\overline{\theta}$	polar body forces
r,z,θ	polar coordinates
t	time, sec
V	volume of liquid, cu in.
α_n	n^{th} root of J_1
μ	dynamic viscosity, centipoises
ν	kinematic viscosity, centistokes
ρ	density of fluid, lb/sq in.

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ω angular velocity, rps

Subscripts:

i initial level

w tank wall

0 at center of tank

Superscript:

* tank bottom exposed

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TABLE I. - EXPERIMENTAL LIQUID LEVELS AT TANK WALL AND CORRESPONDING TIMES

(a) Kinematic viscosity, 1 centistoke

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 16 in.							
0	0	0	0	0	0	0	0
5.0	.20	4.3	.50	5.5	.60	4.7	.30
15.7	.30	11.2	.70	9.3	1.10	7.4	1.00
19.0	.30	22.1	.90	14.5	1.20	12.2	1.80
30.5	.30	30.0	1.10	20.5	1.40	19.3	2.40
40.0	.30	40.0	1.30	26.5	1.70	28.1	3.30
50.0	.40	50.0	1.50	30.0	2.30	35.0	4.20
60.0	.45	60.0	1.70	42.0	2.55	45.0	4.80
75.0	.50	71.5	1.90	50.0	3.00	58.0	5.70
90.0	.60	79.5	2.05	58.0	3.25	75.0	6.40
105.0	.70	94.5	2.20	68.0	3.50	90.0	7.10
119.0	.75	104.5	2.30	78.0	3.80	105.0	7.70
134.0	.80	118.0	2.50	91.0	4.25	120.0	8.20
150.0	.90	134.0	2.60	115.0	4.50	141.0	8.70
165.0	1.00	150.0	2.80	118.0	4.80	160.0	7.20
180.0	1.05	165.0	3.00	132.0	5.10		
200.0	1.10	179.0	3.10	148.0	5.35		
220.0	1.10	200.0	3.20	163.0	5.60		
240.0	1.15	219.0	3.35	177.0	5.80		
270.0	1.20	239.0	3.50	198.0	6.00		
300.0	1.25	270.0	3.65	218.0	6.20		
600.0	1.55	300.0	3.70	237.0	6.40		
		600.0	4.30	272.0	6.65		
				300.0	6.85		
				600.0	7.70		
Initial liquid level, 8 in.							
0	0	0	0	0	0	0	0
5.1	.10	4.3	.20	2.4	.20	5.3	1.00
10.7	.00	9.2	.60	8.0	1.20	9.6	2.40
17.3	.20	15.2	.80	13.6	1.80	16.2	3.50
28.8	.30	22.1	1.20	24.0	2.60	27.6	4.60
50.0	.70	29.0	1.60	30.7	3.10	37.2	5.50
65.0	.85	40.0	1.80	43.6	3.70	44.4	6.20
81.3	1.00	50.9	2.10	50.6	4.00	55.0	6.90
99.4	1.10	64.7	2.40	60.0	4.40	70.8	7.70
119.0	1.20	81.5	2.70	71.6	4.80	81.6	8.15
147.0	1.30	101.0	3.00	82.8	5.20	90.0	8.40
180.0	1.40	125.0	3.25	97.0	5.50	102.5	8.80
213.0	1.45	148.0	3.50	114.0	5.80	112.0	9.10
239.0	1.50	178.0	3.80	124.0	6.00	119.0	9.30
272.0	1.55	221.0	3.90	137.0	6.20	135.0	9.70
300.0	1.55	242.0	4.00	153.0	6.40	150.0	9.90
600.0	1.70	274.0	4.10	167.0	6.55	165.0	[a]10.20
		298.0	4.15	183.0	6.70	180.0	10.35
		600.0	4.40	204.0	6.80	200.0	10.60
				217.0	6.90	220.0	10.80
				232.0	7.05	240.0	10.90
				245.0	7.15	270.0	11.15
				260.0	7.20	300.0	11.25
				279.0	7.30	600.0	11.70
				300.0	7.40		
				600.0	7.80		

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 12 in.							
0	0	0	0	0	0	0	0
4.2	.10	4.2	.30	3.3	.50	2.6	.10
7.5	.10	8.2	.40	5.5	.80	5.4	1.00
10.8	.20	14.2	.70	8.5	1.10	8.4	1.30
15.7	.10	19.1	.80	11.3	1.30	11.4	2.00
22.3	.20	25.0	1.00	17.5	1.40	15.0	2.50
31.5	.25	31.0	1.10	25.0	2.10	21.6	3.00
40.3	.30	41.0	1.40	35.4	2.70	29.4	4.00
55.1	.40	51.8	1.70	48.1	3.20	40.2	4.80
65.6	.50	66.7	2.00	62.2	3.80	54.0	5.80
78.0	.70	79.5	2.30	76.4	4.20	69.6	6.70
90.0	.80	104.0	2.70	98.0	4.85	84.5	7.40
104.0	.85	127.0	2.90	114.0	5.20	113.0	8.30
124.0	.90	151.0	3.10	138.0	5.65	135.0	8.90
147.0	1.00	181.0	3.35	165.0	6.00	167.0	9.60
164.0	1.05	201.0	3.55	202.0	6.40	192.0	10.00
185.0	1.10	239.0	3.75	226.0	6.70	225.0	10.40
205.0	1.15	264.0	3.85	252.0	6.85	241.0	10.60
230.0	1.20	300.0	4.00	300.0	7.10	263.0	10.80
265.0	1.30	600.0	4.40	600.0	7.70	300.0	11.00
300.0	1.40					600.0	12.00
600.0	1.60						
Initial liquid level, 4 in.							
0	0	0	0	0	0	0	0
3.1	.05	6.3	.50	3.3	.85	4.3	.30
9.7	.25	9.3	1.10	6.3	1.00	6.1	1.65
16.3	.35	13.2	1.00	8.5	1.70	8.5	2.50
24.5	.55	19.2	1.70	11.5	2.60	12.1	3.90
37.6	.70	24.1	2.00	16.7	2.90	16.7	4.50
44.2	.75	31.1	2.30	21.2	3.35	21.1	5.20
53.4	.90	43.0	2.70	30.9	4.20	30.0	[a]6.50
60.6	1.00	57.8	3.15	46.8	5.10	41.4	7.35
75.4	1.10	68.7	3.30	68.8	[a]5.90	61.2	8.10
90.0	1.25	99.0	3.75	95.6	6.45	76.2	8.55
105.0	1.35	118.0	4.00	122.0	6.75	91.0	8.85
118.0	1.45	139.0	4.15	169.0	7.10	106.0	9.10
136.0	1.45	160.0	4.20	197.0	7.15	123.0	9.30
148.0	1.50	180.0	4.30	230.0	7.20	150.0	9.55
164.0	1.55	210.0	[a]4.35	257.0	7.25	187.0	9.65
179.0	1.55	244.0	4.45	300.0	7.30	203.0	9.75
200.0	1.60	300.0	4.50			224.0	9.80
220.0	1.60					246.0	9.85
240.0	1.60					300.0	9.90
300.0	1.60					600.0	9.95
600.0	1.65						

[a]Tank bottom becomes exposed.

TABLE I. - Continued. EXPERIMENTAL LIQUID LEVELS

AT TANK WALL AND CORRESPONDING TIMES

(b) Kinematic viscosity, 3.4 centistokes

Tangential velocity, ωR, in./sec					
83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 16 in.					
0	0	0	0	0	0
5.4	.40	8.7	1.30	2.6	.60
15.4	.80	23.0	1.80	15.5	3.50
29.5	1.20	29.2	2.20	26.6	3.80
45.6	1.60	44.8	3.00	37.1	5.70
59.6	2.00	58.8	3.60	56.1	6.00
74.7	2.30	74.3	4.20	71.1	6.90
88.0	2.60	89.1	4.60	85.9	7.60
104.0	2.80	104.0	5.00	101.0	8.20
120.0	3.10	118.0	5.30	116.0	8.70
150.0	3.30	149.0	5.90	133.0	9.10
180.0	3.60	178.0	6.20		
218.0	3.80	209.0	6.50		
241.0	4.00	238.0	6.70		
300.0	4.10	300.0	7.00		

TABLE I. - Continued. EXPERIMENTAL LIQUID LEVELS AT TANK WALL AND CORRESPONDING TIMES

(c) Kinematic viscosity, 34.5 centistokes

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 16 in.							
0	0	0	0	0	0	0	0
2.5	.10	1.2	.20	1.0	.10	1.0	.10
10.8	.30	3.3	.60	4.9	1.00	2.0	.70
22.3	.60	13.6	1.30	9.8	1.90	7.6	2.60
35.4	.90	28.0	2.10	17.4	3.00	14.4	4.30
48.6	1.10	43.4	2.90	25.1	4.00	21.2	5.70
60.1	1.20	55.7	3.30	38.0	5.10	28.6	6.90
71.6	1.30	74.3	3.60	48.7	5.80	39.1	8.10
89.7	1.40	89.6	3.80	61.7	6.40	49.9	9.10
106.0	1.50	104.0	4.00	73.1	6.80		
121.0	1.60	119.0	4.10	87.7	7.10		
136.0	1.60	134.0	4.10	101.0	7.30		
151.0	1.70	147.0	4.20	147.0	7.50		
Initial liquid level, 8 in.							
0	0	0	0	0	0	0	0
2.5	.10	5.3	1.10	1.0	.15	1.4	1.10
5.8	.30	16.3	2.50	3.6	1.60	7.9	7.60
14.1	.70	26.2	3.30	10.4	3.20	18.6	9.60
27.2	1.20	41.2	3.90	20.4	4.90	29.3	[a]10.50
38.7	1.40	52.1	4.10	31.1	6.00	37.6	11.30
48.6	1.50	73.1	4.30	42.6	6.70	49.4	11.70
60.1	1.60	90.0	4.40	57.8	7.10	61.9	11.80
74.9	1.70	126.0	4.50	72.4	7.30	82.2	11.90
119.0	1.70			87.7	7.40	123.0	12.00
				102.0	7.50		

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 12 in.							
0	0	0	0	0	0	0	0
2.5	.20	3.3	.60	1.0	.20	3.2	1.70
4.1	.30	14.3	1.80	2.9	1.10	12.8	4.80
9.1	.50	28.3	2.90	9.7	2.40	21.9	6.90
15.6	.70	43.2	3.50	18.2	3.70	31.6	8.50
27.1	.90	58.2	4.00	27.9	5.00	41.8	9.60
40.3	1.20	75.1	4.20	42.3	6.10	54.1	10.50
55.1	1.40	89.0	4.30	52.2	6.60	58.7	10.80
74.8	1.50	149.0	4.50	61.3	7.00	69.0	11.30
89.6	1.60			73.6	7.20	78.1	11.50
104.0	1.60			86.8	7.40	87.0	11.70
121.0	1.70			101.0	7.50	97.0	11.80
149.0	1.70			146.0	7.70	147.0	12.10
Initial liquid level, 4 in.							
0	0	0	0	0	0	0	0
2.5	.30	2.3	.60	3.6	2.20	1.3	2.00
10.8	.80	10.3	1.50	8.8	4.10	7.4	5.50
17.2	1.20	17.3	3.40	17.1	[a]5.90	11.0	[a]7.10
25.5	1.40	25.2	4.00	23.9	6.60	18.2	8.80
35.4	1.60	35.2	4.30	32.2	7.00	26.6	9.50
42.0	1.70	48.1	4.40	40.5	7.10	33.3	9.70
		59.1	[a]4.50	48.1	7.20	42.3	9.90
				58.6	7.20	60.4	10.00

[a]Tank bottom becomes exposed.

TABLE I. - Continued. EXPERIMENTAL LIQUID LEVELS AT TANK WALL AND CORRESPONDING TIMES

(d) Kinematic viscosity, 91 centistokes

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 16 in.							
0	0	0	0	0	0	0	0
2.5	.20	1.4	.30	1.0	.20	1.5	1.10
7.5	.40	6.4	1.20	3.6	1.40	4.0	2.60
10.8	.60	11.4	1.90	7.4	2.60	6.4	3.60
17.4	.80	18.4	2.60	11.2	3.50	10.1	5.00
25.6	1.00	22.4	2.90	16.5	4.30	14.4	6.30
35.4	1.20	29.4	3.30	21.7	5.00	19.9	7.70
43.6	1.30	34.5	3.50	28.7	5.90	25.5	8.70
50.2	1.40	40.5	3.80	37.2	6.50		
58.4	1.50	46.5	3.90	45.4	6.90		
66.6	1.50	51.5	4.00	54.5	7.20		
		57.5	4.10	67.4	7.40		
		64.5	4.20				
		69.5	4.20				
Initial liquid level, 8 in.							
0	0	0	0	0	0	0	0
2.5	.20	1.4	.40	.6	.30	.8	.80
4.2	.40	4.3	1.40	1.4	1.00	2.7	2.80
9.1	.80	8.3	2.30	3.0	1.90	5.7	4.70
14.1	1.00	13.3	3.00	4.5	2.50	9.3	6.60
19.0	1.30	17.2	3.40	6.0	3.00	12.9	7.90
23.9	1.40	22.2	3.80	7.6	3.60	17.7	9.20
28.9	1.50	26.1	4.00	9.8	4.20	21.9	10.00
33.8	1.50	31.1	4.20	12.9	5.00	26.7	[a]10.70
38.7	1.60	36.1	4.30	16.7	5.70	32.1	11.10
43.6	1.60	41.0	4.40	21.3	6.30	36.3	11.30
48.6	1.70	45.0	4.40	28.1	6.80	43.0	11.50
60.0	1.70	60.0	4.50	35.8	7.20	56.8	11.60
				44.9	7.40		
				58.6	7.50		

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 12 in.							
0	0	0	0	0	0	0	0
2.6	.20	1.4	.40	.6	.20	1.6	1.20
7.5	.60	4.4	1.00	2.2	.30	4.0	3.00
15.7	1.00	8.4	1.90	4.5	2.00	7.6	4.80
22.3	1.20	12.4	2.40	8.2	3.20	11.8	6.40
28.9	1.30	17.4	2.90	12.7	4.40	16.0	7.70
35.4	1.40	23.4	3.30	16.4	5.00	20.8	8.90
40.4	1.50	29.4	3.70	20.9	5.70	27.4	10.00
45.3	1.50	33.4	3.90	25.4	6.20	34.1	10.80
51.9	1.60	40.3	4.00	33.6	6.90	40.1	11.20
56.8	1.60	43.3	4.10	41.0	7.30	47.3	11.50
70.0	1.70	53.3	4.20	47.8	7.50	56.9	11.80
		62.3	4.30	55.2	7.60	63.5	11.90
		67.3	4.40	67.2	7.70		
Initial liquid level, 4 in.							
0	0	0	0	0	0	0	0
2.5	.30	1.3	.50	.5	.20	.5	1.00
4.2	.50	2.3	1.00	2.1	1.80	2.5	3.60
5.8	.70	3.4	1.40	3.6	2.90	4.5	5.20
7.5	.90	5.5	2.00	5.9	4.00	6.5	[a]6.40
10.8	1.10	7.5	2.50	8.2	4.90	8.8	7.60
14.1	1.30	9.6	2.90	9.7	[a]5.40	10.0	8.00
17.3	1.50	12.8	3.30	12.0	6.00	12.4	8.60
22.3	1.60	15.9	3.60	14.3	6.30	14.8	9.00
25.6	1.60	18.0	3.80	16.6	6.60	18.5	9.30
30.5	1.70	22.2	3.90	18.1	6.70	22.4	9.50
		26.3	4.00	21.2	6.80	24.6	9.60
		30.5	4.10	24.2	6.90	41.3	9.70
		52.5	4.20	30.3	7.00		
				58.5	7.10		

[a]Tank bottom becomes exposed.

TABLE I. - Concluded. EXPERIMENTAL LIQUID LEVELS AT TANK WALL AND CORRESPONDING TIMES

(e) Kinematic viscosity, 260 centistokes

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 16 in.							
0	0	0	0	0	0	0	0
2.4	.30	1.9	.50	.2	.30	.1	.30
4.1	.45	2.9	.90	1.1	1.10	.4	.90
5.8	.65	3.9	1.25	1.8	1.90	1.2	1.70
7.4	.80	4.9	1.50	2.6	2.25	1.8	2.70
9.1	.90	6.0	1.80	3.3	2.60	2.4	3.40
10.7	1.00	7.0	2.00	4.1	2.95	3.0	4.00
12.3	1.10	8.0	2.20	4.8	3.35	4.1	4.90
14.0	1.20	9.0	2.40	5.6	3.60	5.3	5.80
15.6	1.25	11.0	2.70	6.3	3.95	6.5	6.50
17.3	1.30	13.1	3.00	7.1	4.20	7.7	7.20
18.9	1.35	15.1	3.25	7.8	4.45	8.9	7.80
20.6	1.40	18.2	3.55	9.3	4.90	9.5	8.20
22.2	1.45	23.3	3.75	9.8	5.40	11.8	9.00
23.8	1.50	26.3	3.90	12.3	5.75	15.3	10.10
25.5	1.55	29.3	4.05	13.8	6.10		
27.1	1.55			16.1	6.40		
28.8	1.60			18.3	6.80		
30.4	1.65			20.5	7.00		
				24.3	7.40		
				27.3	7.60		
Initial liquid level, 8 in.							
0	0	0	0	0	0	0	0
2.5	.30	.9	.60	.2	.40	1.6	2.20
4.2	.60	2.8	1.60	1.8	2.30	2.8	4.10
7.5	.90	4.8	2.30	3.3	3.20	4.5	5.90
10.8	1.20	6.7	2.90	4.7	4.10	6.3	7.30
14.1	1.40	8.6	3.30	7.0	5.10	8.1	8.40
17.3	1.50	10.6	3.60	9.2	5.90	9.8	9.30
20.6	1.60	12.5	3.90	11.4	6.50	12.2	[a]10.20
22.3	1.60	14.5	4.10	13.6	6.90	14.0	10.80
25.5	1.60	17.4	4.30	15.1	7.20	15.7	11.20
27.2	1.70	19.3	4.40	17.3	7.40	17.5	11.40
28.8	1.70	21.2	4.50	18.8	7.60	19.3	11.60
		24.1	4.50	21.0	7.70	21.0	11.80
		26.1	4.60	23.2	7.70	25.7	12.00
		28.0	4.60	28.5	7.80		

Tangential velocity, ωR, in./sec							
51.36		83.28		111		138.7	
Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.	Time, t, sec	Liquid-level rise, Δh_w, in.
Initial liquid level, 12 in.							
0	0	0	0	0	0	0	0
2.4	.30	1.9	.40	.5	.30	.9	.90
4.6	.60	2.9	1.20	2.1	1.90	2.2	2.80
5.7	.80	3.9	1.40	3.6	2.80	4.0	4.90
7.4	.90	4.9	1.70	5.0	3.50	5.2	5.80
9.0	1.00	5.8	2.00	7.2	4.50	6.9	7.00
10.6	1.10	6.8	2.20	8.7	5.00	8.7	8.10
13.9	1.30	7.8	2.50	10.1	5.50	10.4	9.00
17.2	1.40	8.8	2.80	12.3	6.10	12.1	9.80
20.5	1.50	10.8	3.10	14.4	6.50	13.9	10.40
23.8	1.60	12.8	3.40	15.8	6.80	15.6	10.80
27.5	1.60	14.7	3.70	18.1	7.10	19.1	11.70
30.4	1.70	17.7	3.90	20.2	7.40	23.8	12.10
		22.6	4.20	22.4	7.60	27.2	12.40
		24.6	4.30	24.6	7.70		
Initial liquid level, 4 in.							
0	0	0	0	0	0	0	0
2.5	.50	.6	.60	1.0	1.80	.6	1.60
4.2	.80	1.3	1.10	1.7	2.80	1.9	4.30
5.8	1.10	2.4	2.10	3.2	4.10	3.1	[a]5.90
7.5	1.30	3.4	2.50	5.5	[a]5.50	4.4	7.10
9.1	1.40	4.4	2.80	7.8	6.30	5.6	7.90
10.8	1.50	6.5	3.40	9.4	6.70	8.0	8.90
12.4	1.60	7.5	3.60	10.9	6.90	9.2	9.20
13.0	1.60	10.6	4.00	12.4	7.10	11.7	9.50
15.7	1.70	13.7	4.20	14.7	7.20	12.9	9.70
17.3	1.70	19.8	[a]4.40	17.0	7.30		

[a]Tank bottom becomes exposed.

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CA-3 back

TABLE II. - THEORETICAL VALUES OF
STEADY-STATE LIQUID LEVEL RISE
CALCULATED FROM EQUATION (5)

Initial liquid level, H_i, in.	Tangential velocity, ωR, in./sec			
	51.36	83.28	111	138.7
	Steady-state liquid-level rise, ΔH_w, in.			
4	1.7	[a]4.42	[a]7.3	[a]10.0
8	1.7	4.48	7.8	[a]11.9
12	1.7	4.48	7.8	[a]12.4
16	1.7	4.48	7.8	12.4

[a]Tank bottom exposed.

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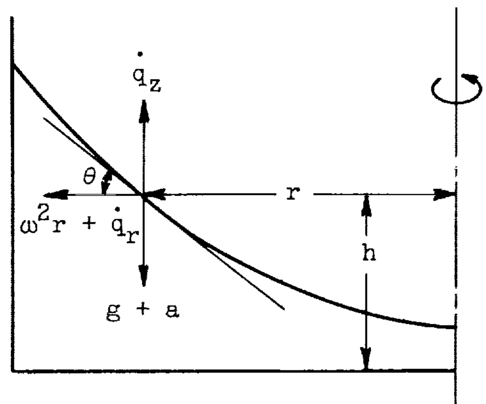
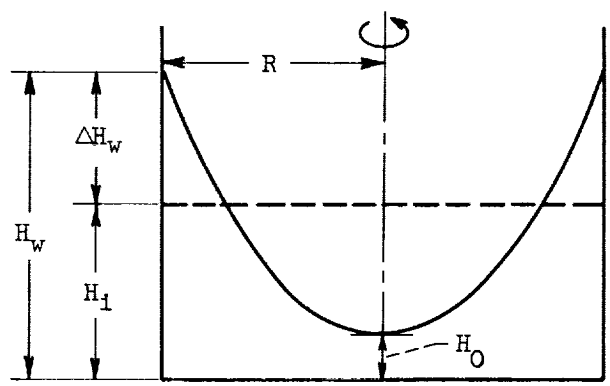
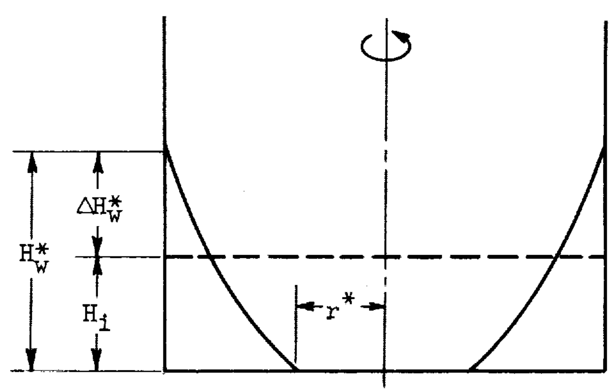


(a) Force balance at free surface.



(b) Steady-state configuration for tank with unexposed bottom.



(c) Steady-state configuration for tank with exposed bottom.

Figure 1. - Fluid-surface configurations in spinning cylindrical tank.

E-877

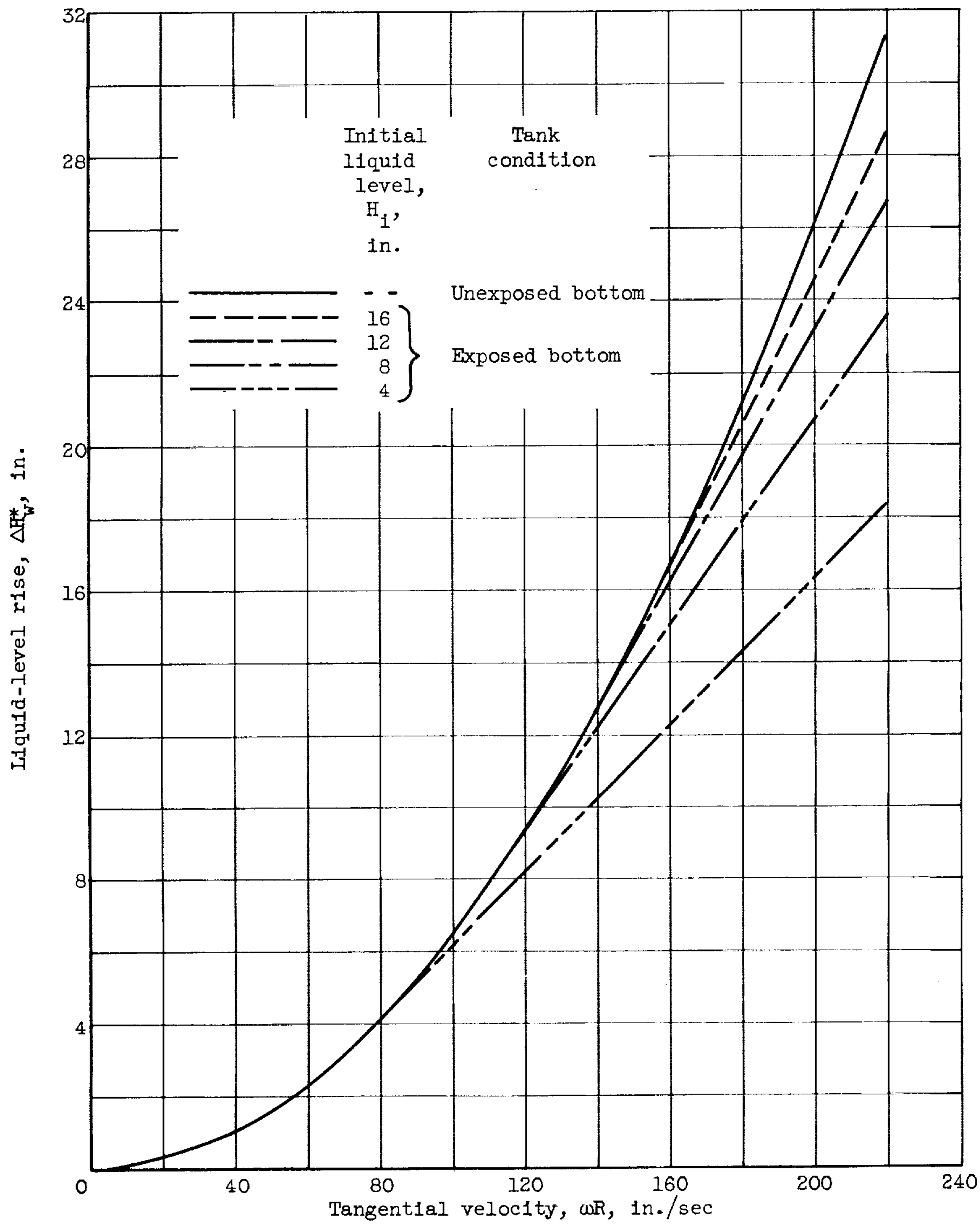
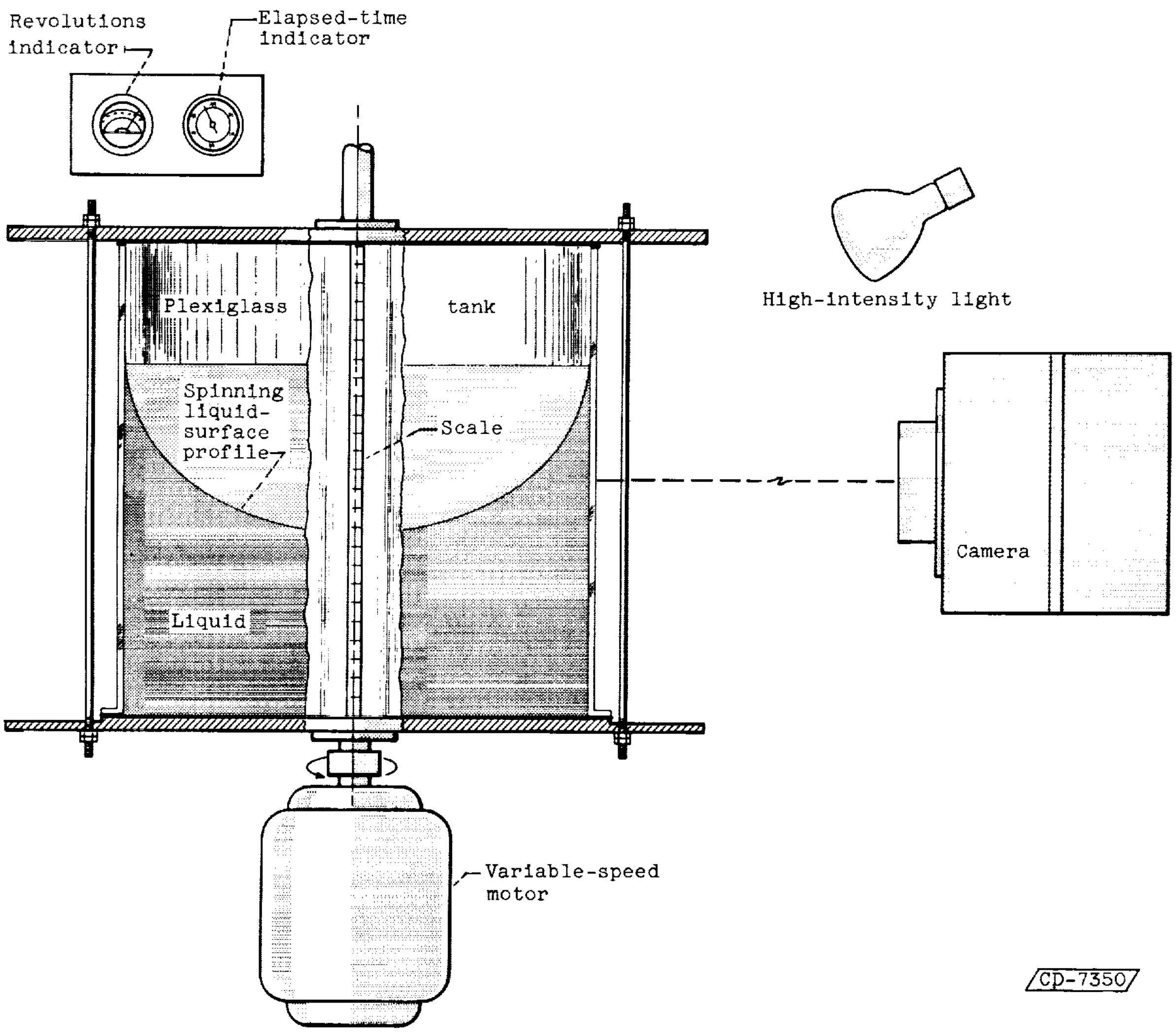


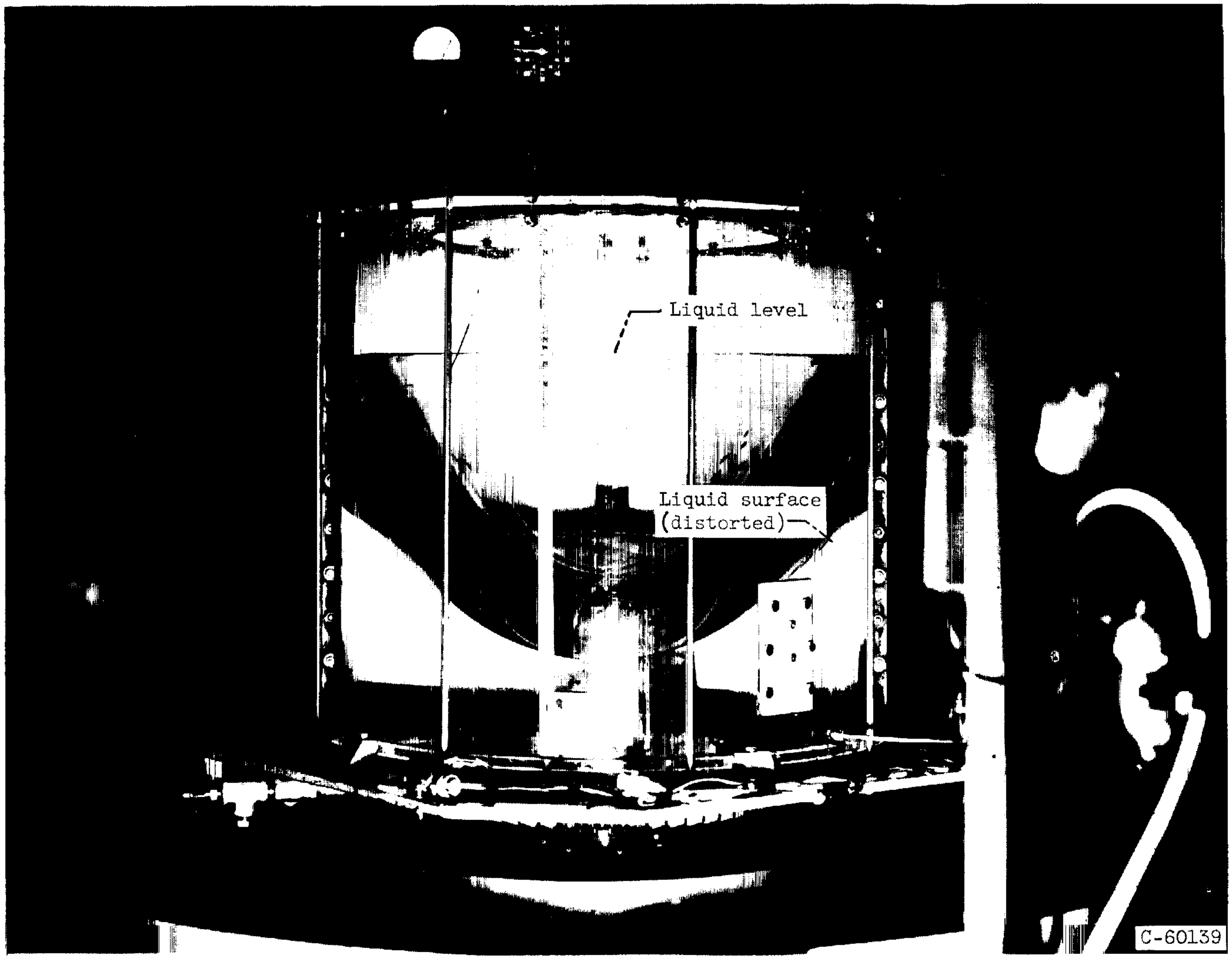
Figure 2. - Influence of initial liquid level on steady state liquid-level rise.

E-877



(a) Schematic diagram of apparatus.

Figure 3. - Spinning-tank test rig.



(b) Photograph of liquid motion.

Figure 3. - Spinning-tank test rig.

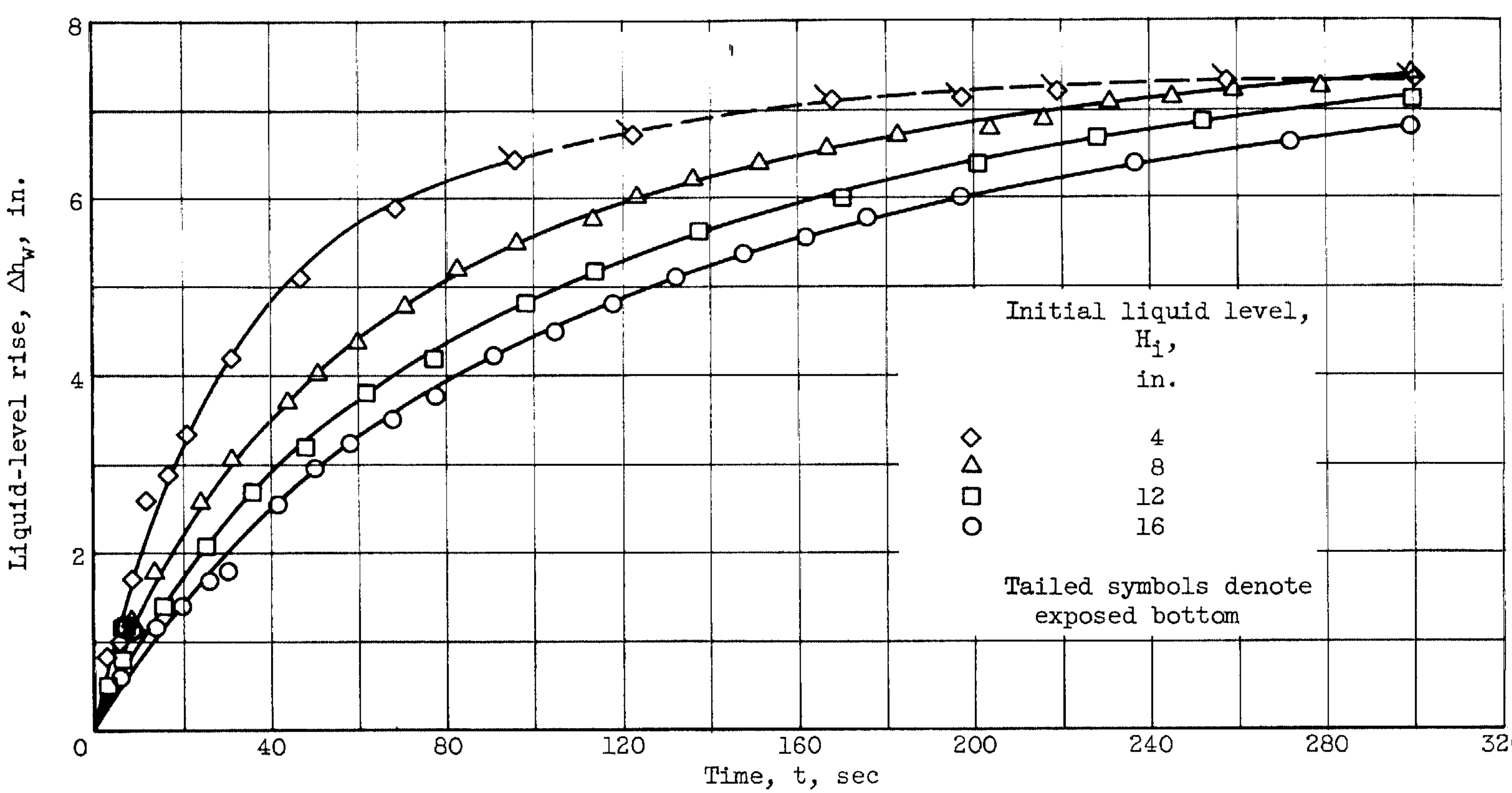
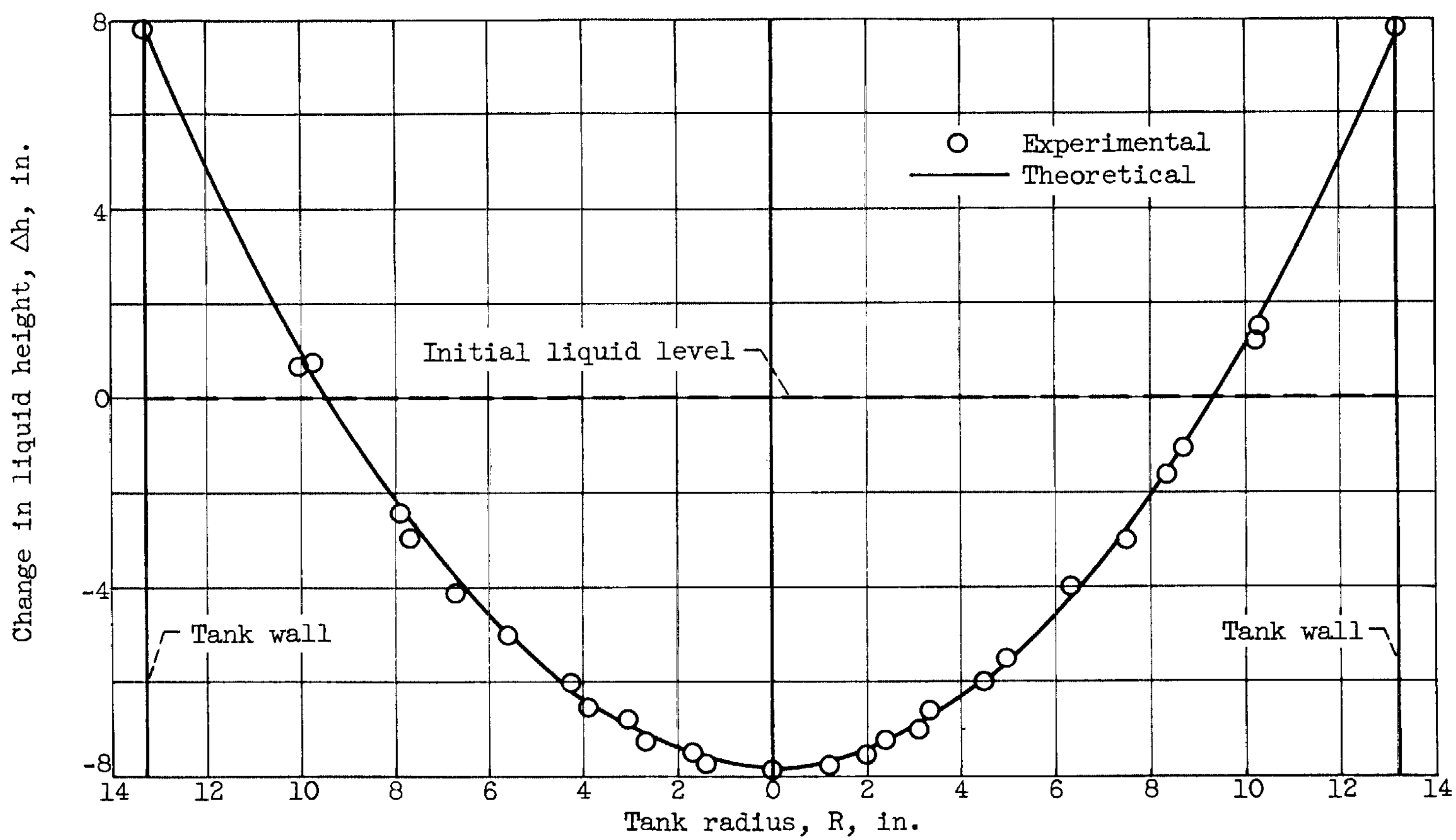
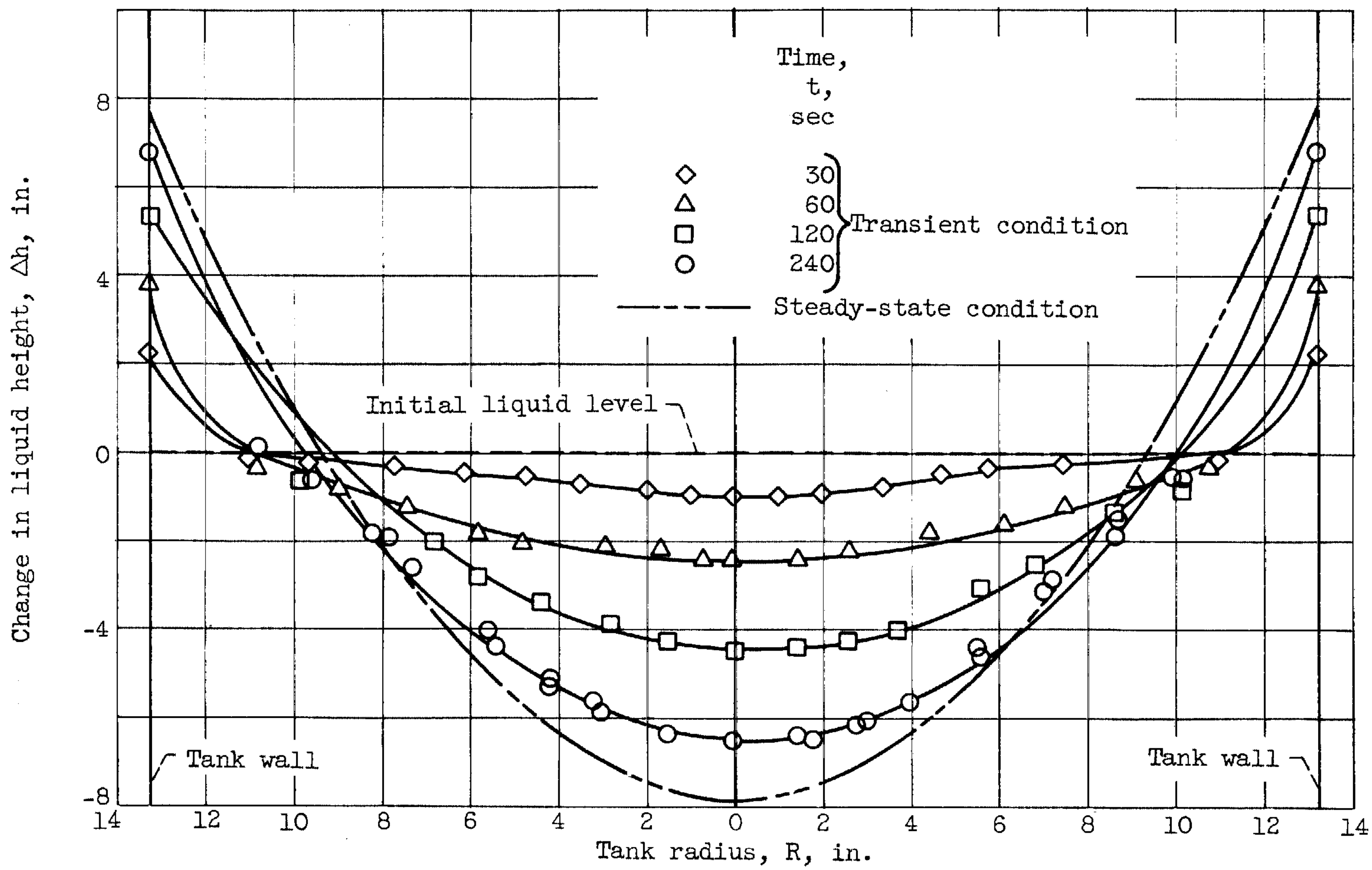


Figure 4. - Liquid-level rise for several values of initial liquid level. Kinematic viscosity, 1 centistoke; tangential velocity, 111 inches per second.



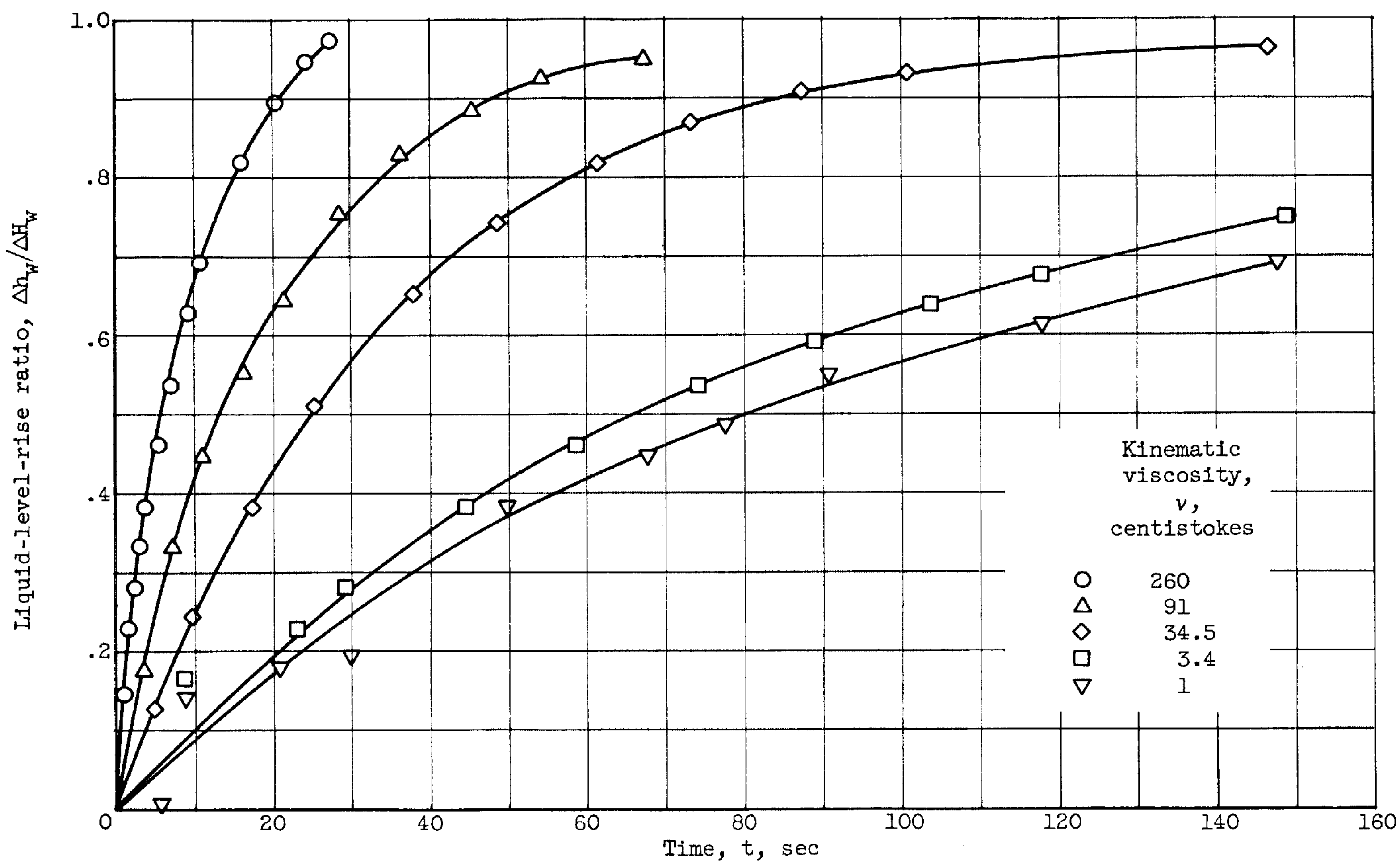
(a) Comparison of steady-state theoretical and experimental free-surface contours.

Figure 5. - Surface profiles. Kinematic viscosity, 1 centistoke; tangential velocity, 111 inches per second; initial steady-state liquid level, 12 inches.



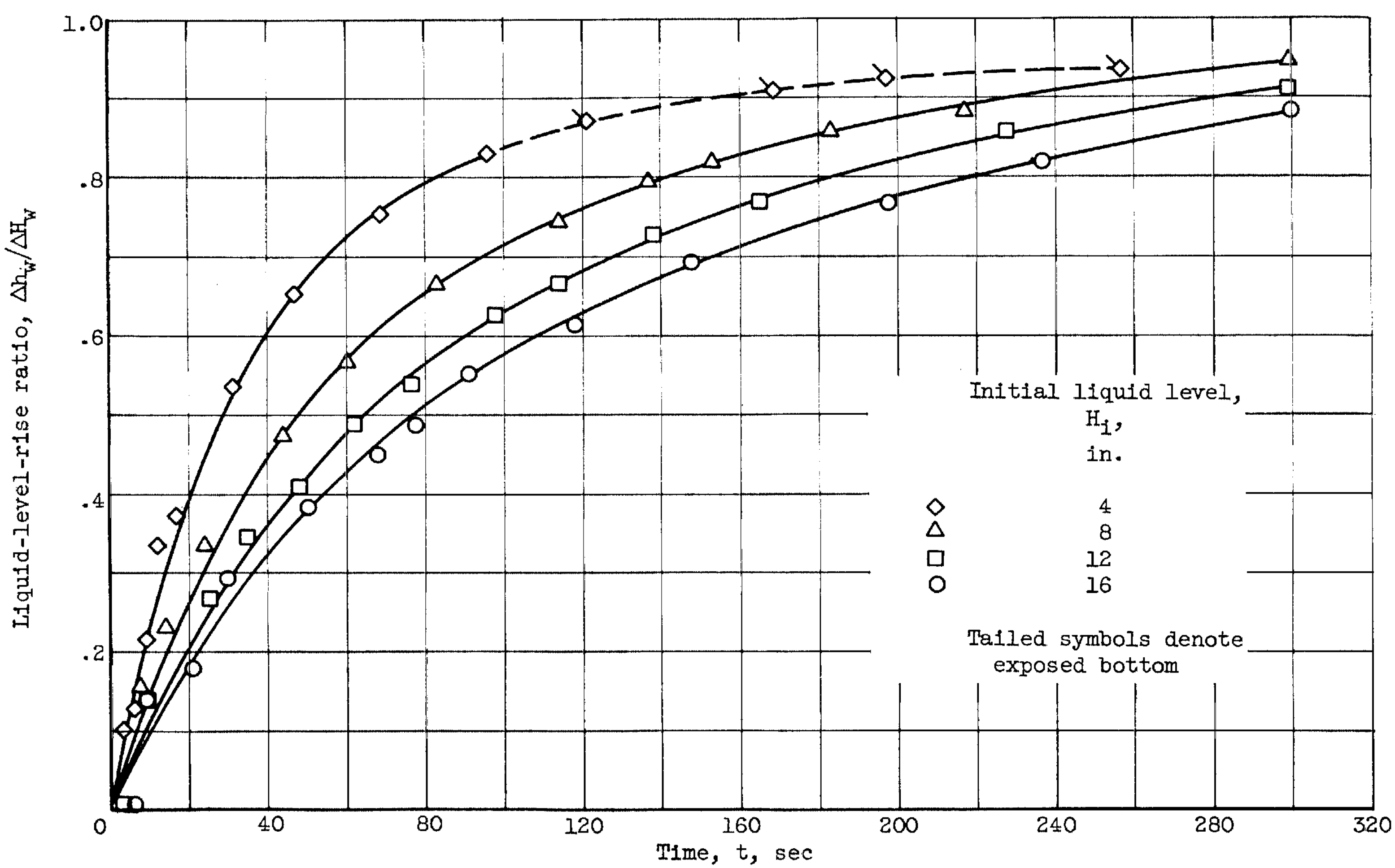
(b) Variation of liquid surface during transient period.

Figure 5. - Concluded. Surface profiles. Kinematic viscosity, 1 centistoke; tangential velocity, 111 inches per second; initial steady-state liquid level, 12 inches.



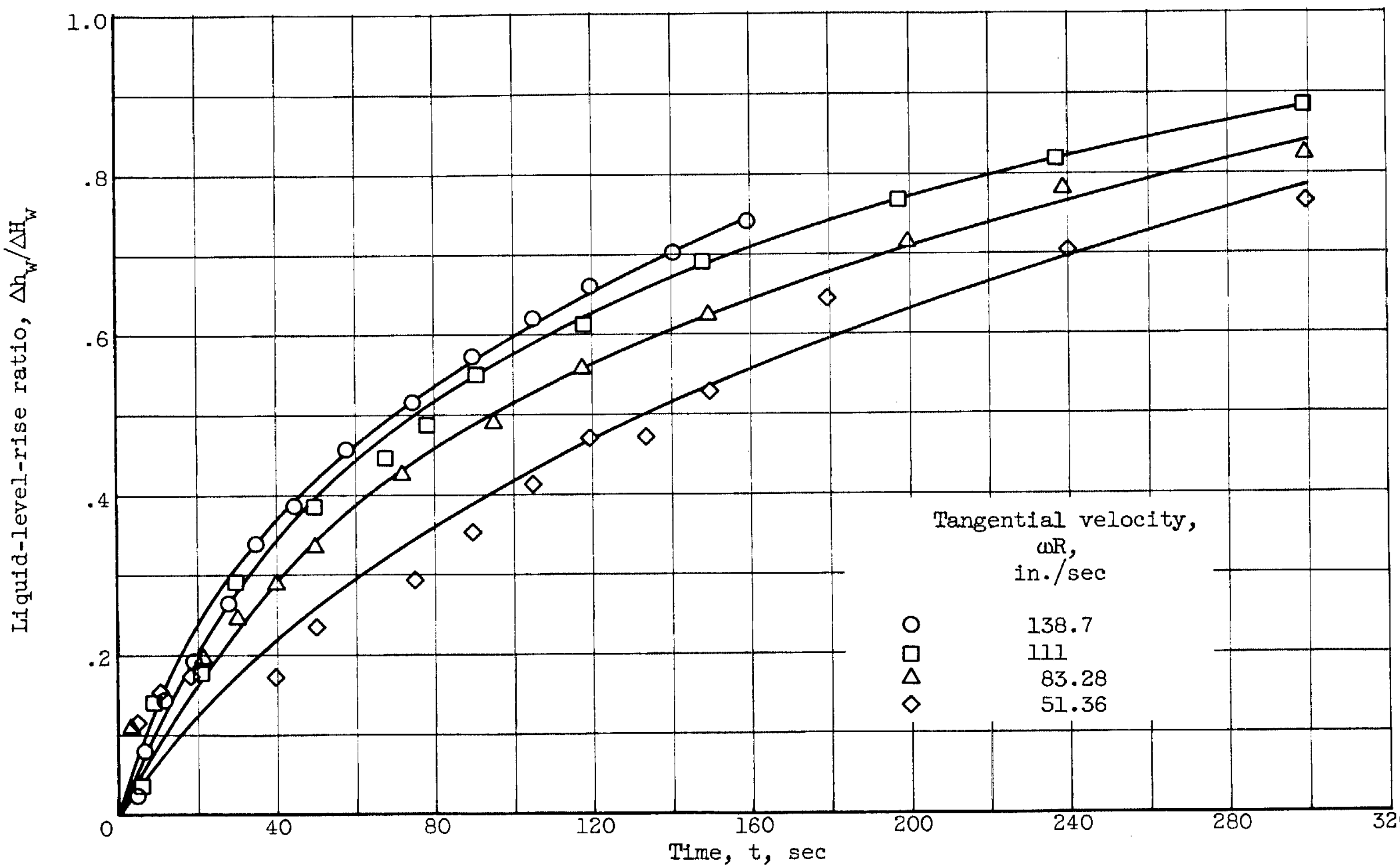
(a) Kinematic viscosity. Tangential velocity, 111 inches per second; initial steady-state liquid level, 16 inches.

Figure 6. - Effects of various conditions on liquid-level-rise ratio.



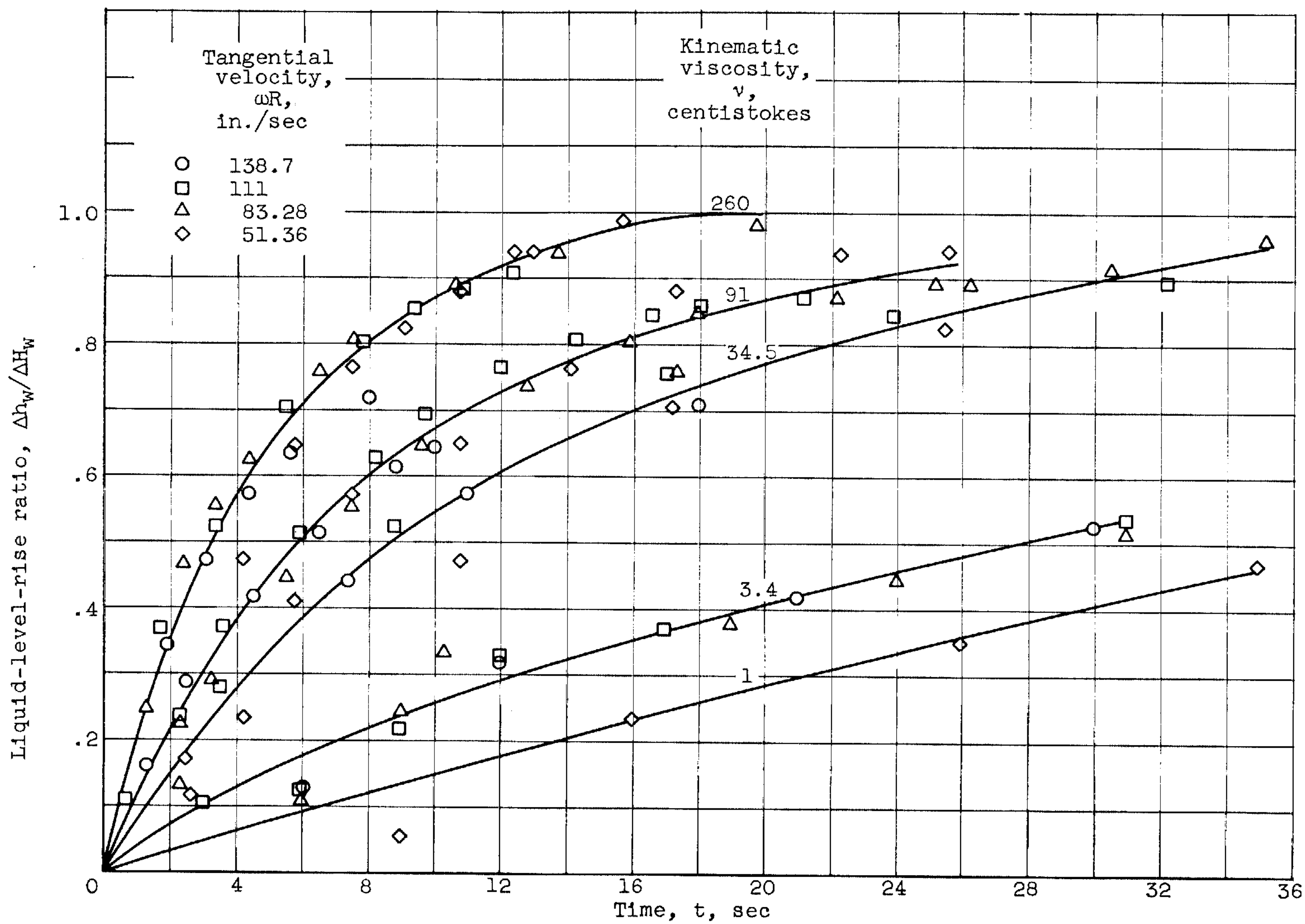
(b) Initial liquid level. Tangential velocity, 111 inches per second; kinetic viscosity, 1 centistoke.

Figure 6. - Continued. Effects of various conditions on liquid-level-rise ratio.



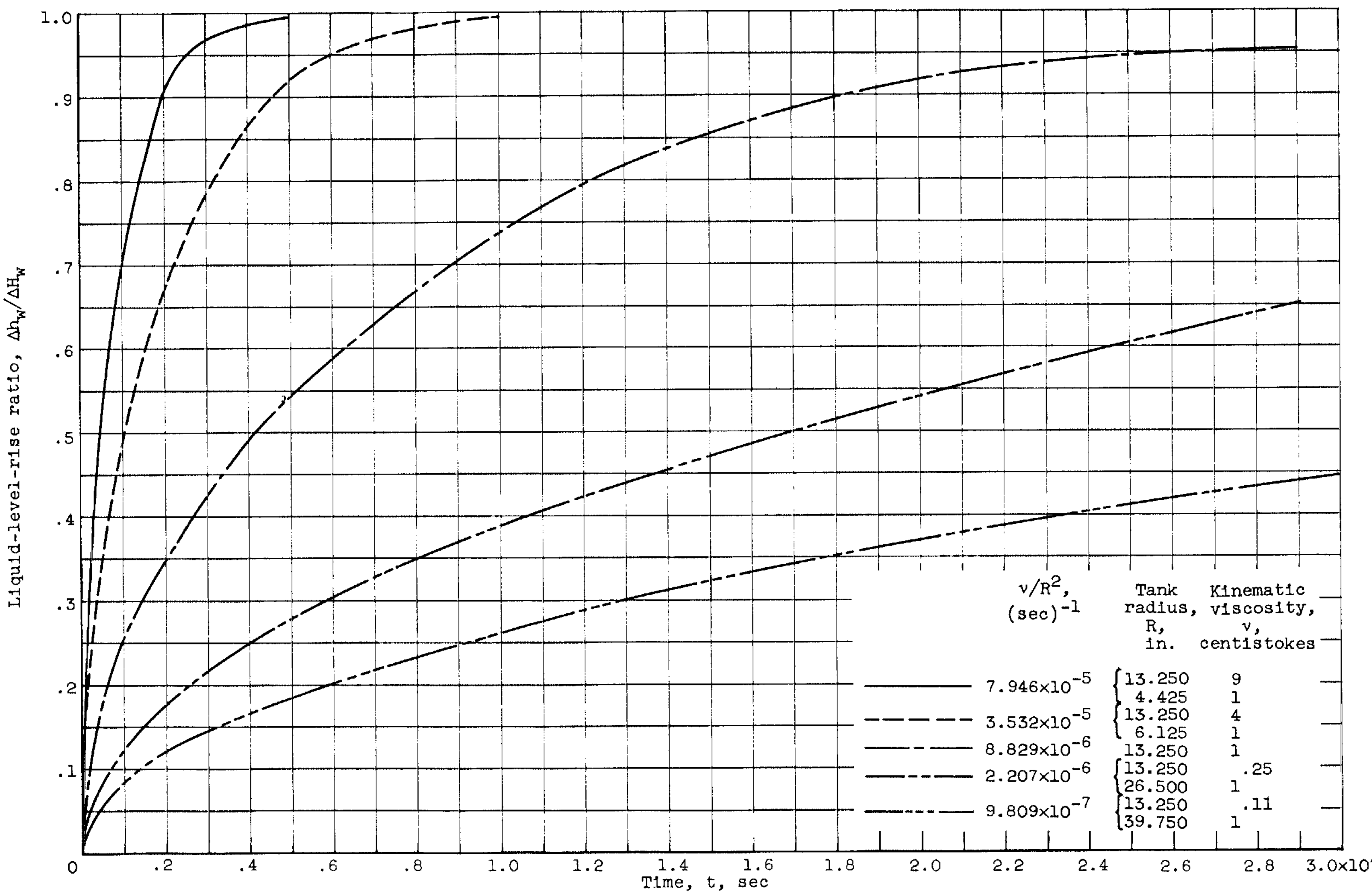
(c) Tangential velocity. Kinematic viscosity, 1 centistoke; initial steady-state liquid level, 16 inches.

Figure 6. - Continued. Effects of various conditions on liquid-level-rise ratio.



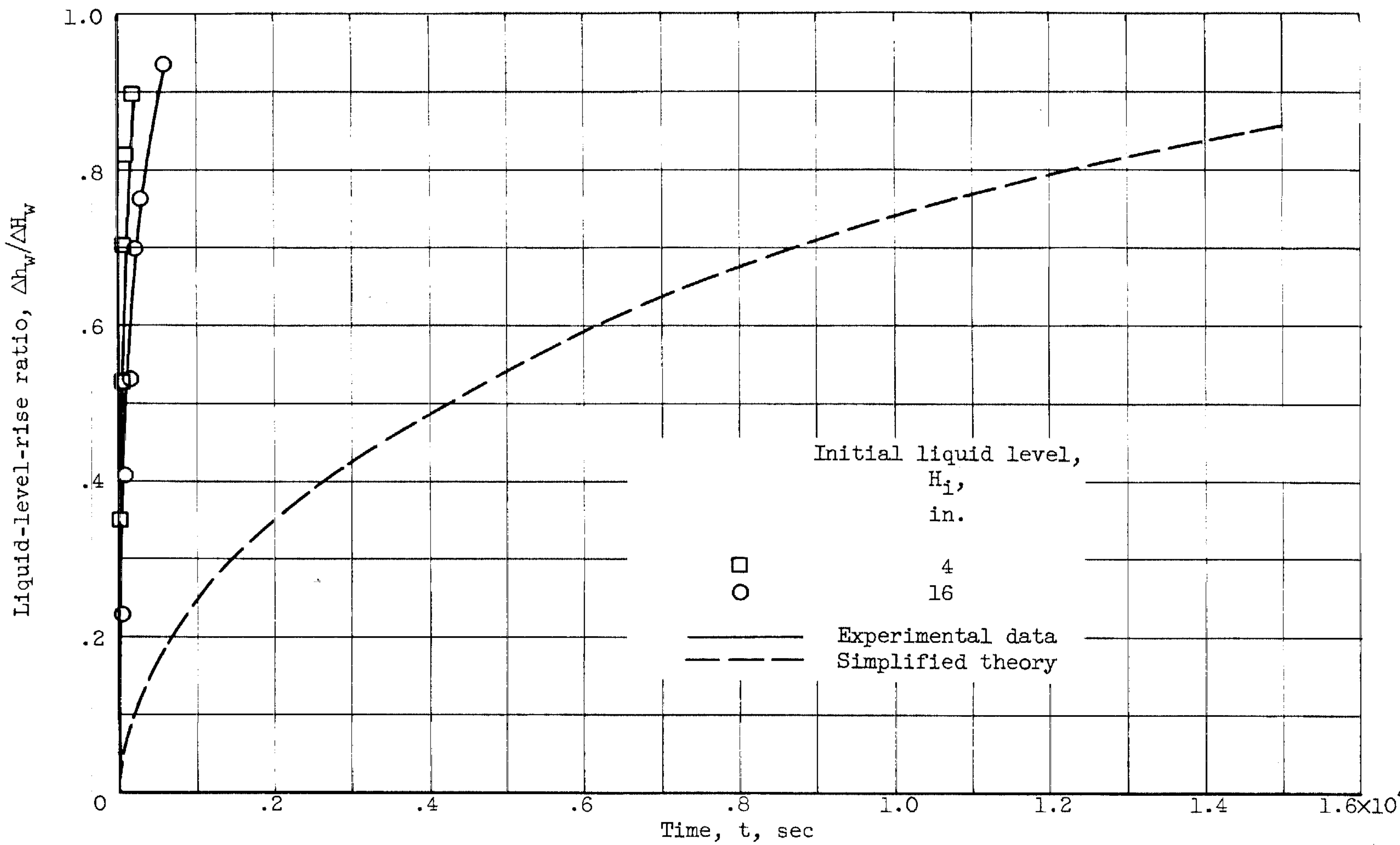
(d) Tangential velocity and kinematic viscosity. Initial steady-state liquid level, 4 inches.

Figure 6. - Continued. Effects of various conditions on liquid-level-rise ratio.



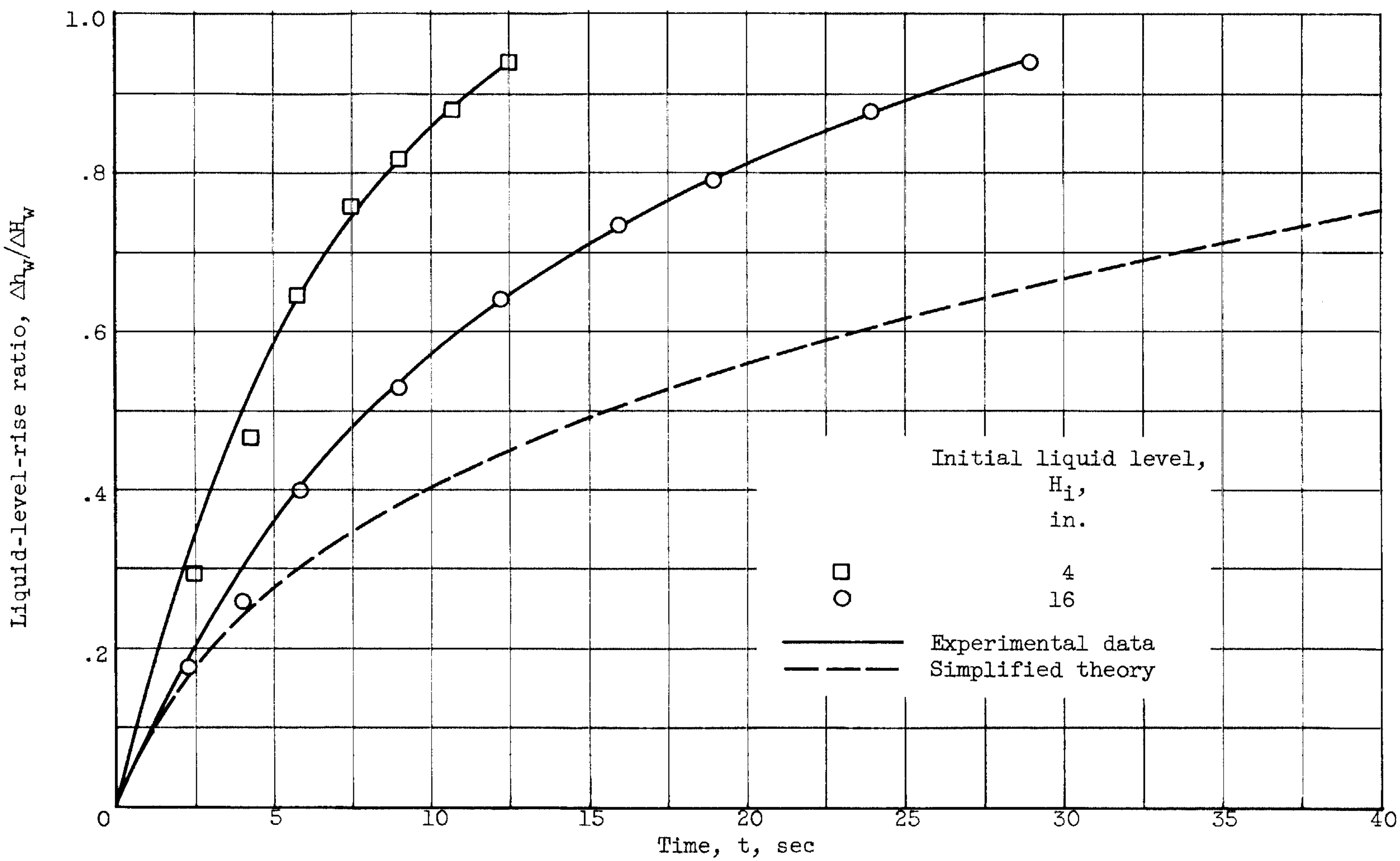
(e) Kinematic viscosity and radius, based on simplified theory.

Figure 6. - Concluded. Effects of various conditions on liquid-level-rise ratio.



(a) Kinematic viscosity, 1 centistoke.

Figure 7. - Comparison of experimental and theoretical liquid-level rise for transient condition. Tank radius, 13.25 inches; tangential velocity, 51.36 inches per second.



(b) Kinematic viscosity, 260 centistokes.

Figure 7. - Concluded. Comparison of experimental and theoretical liquid-level rise for transient condition. Tank radius, 13.25 inches; tangential velocity, 51.36 inches per second.

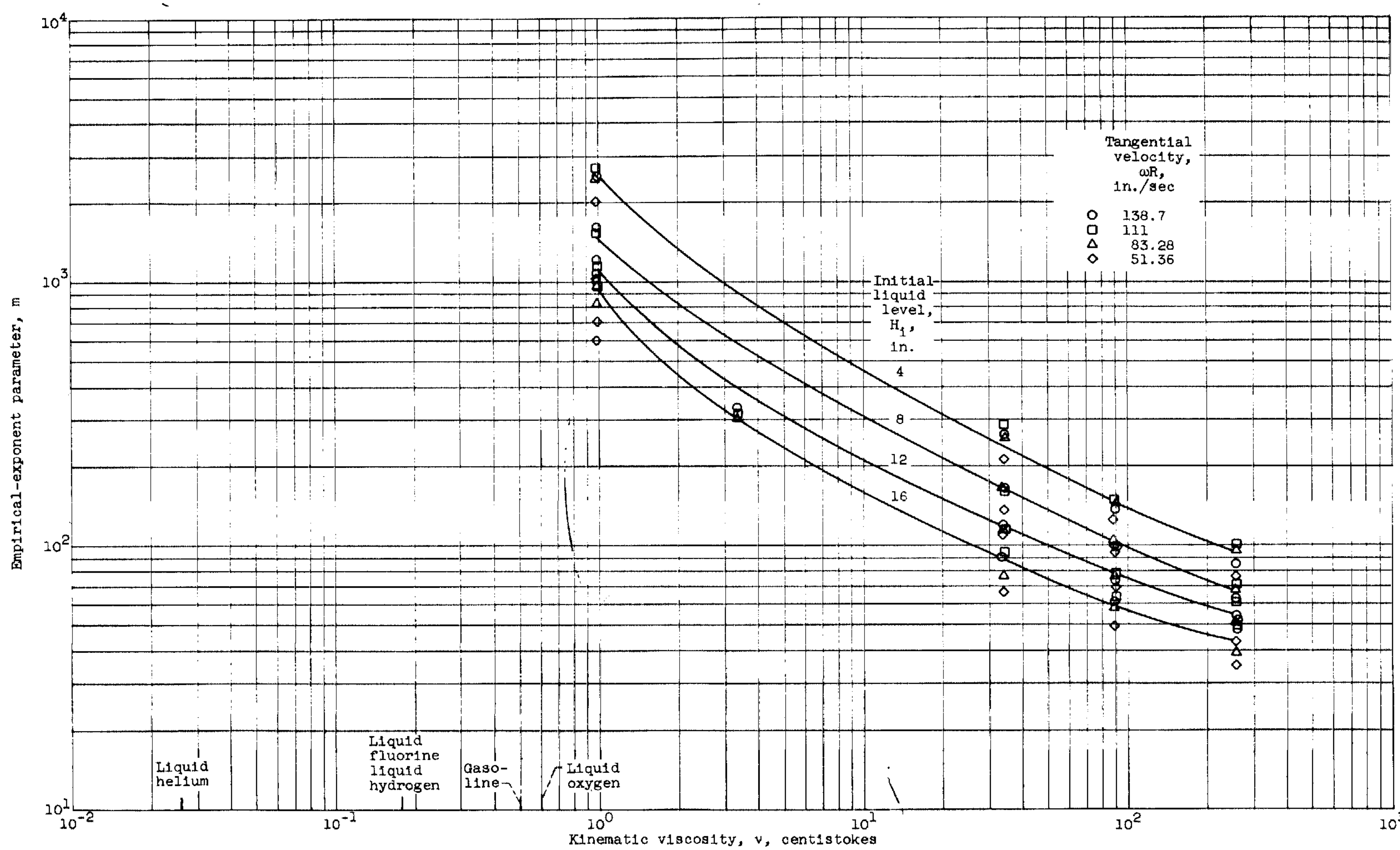
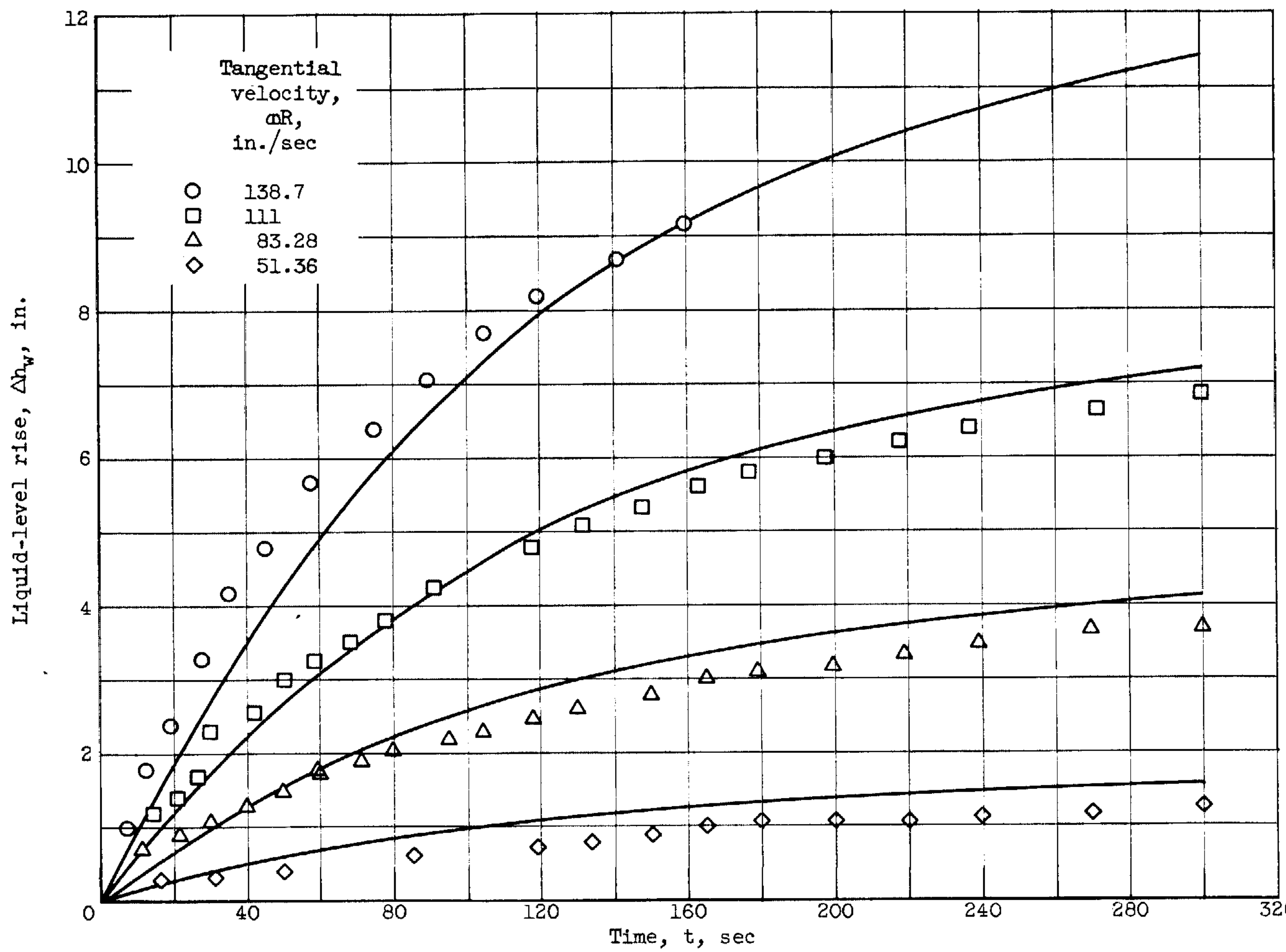
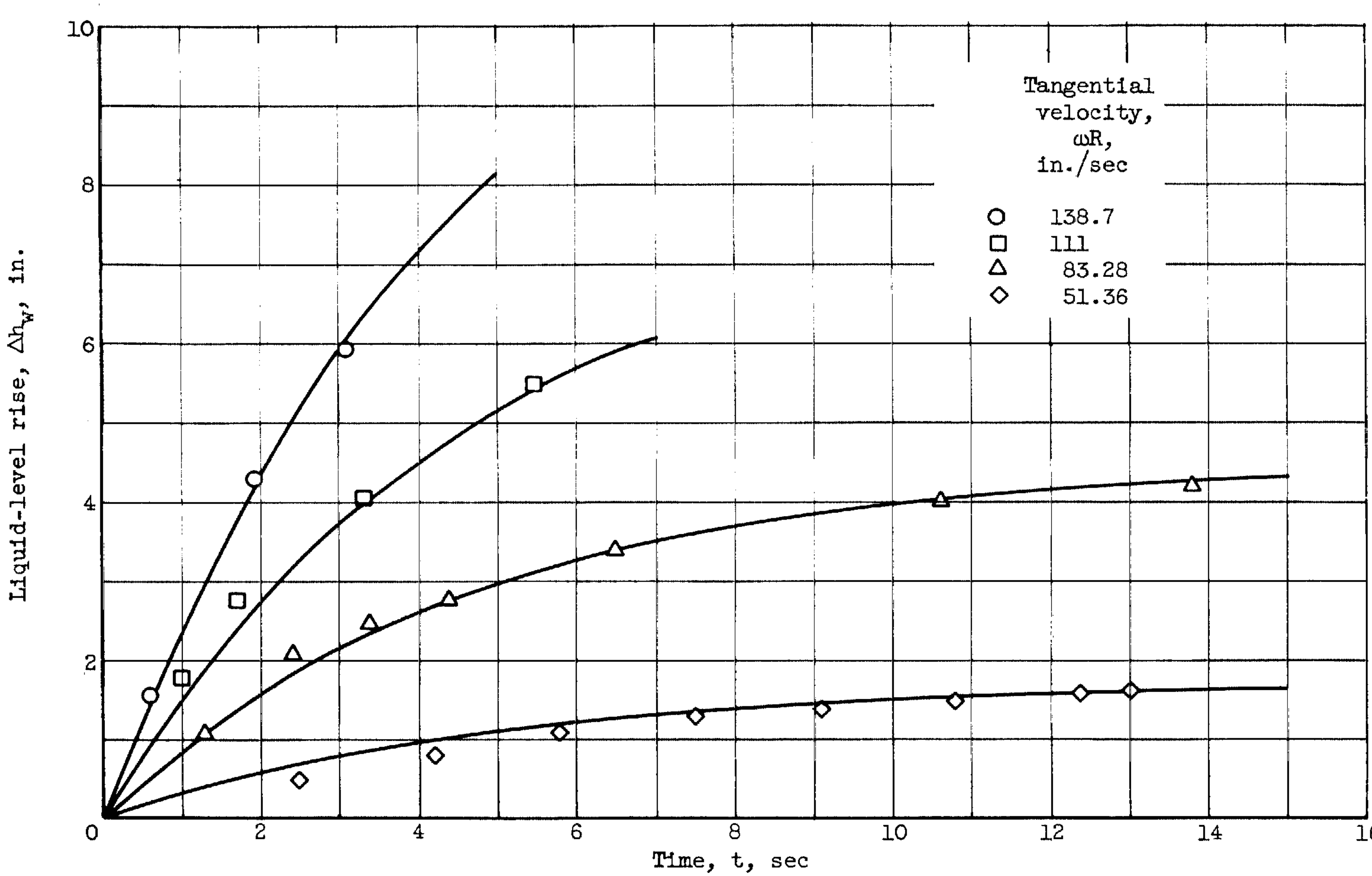


Figure 8. - Variation of parameter m with kinematic viscosity.



(a) Kinematic viscosity, 1 centistoke; initial liquid level, 16 inches.

Figure 9. - Comparison of empirical correlation $\Delta h_w/\Delta H_w = 1 - e^{-m \frac{\nu t}{R^2}}$ and experimental data.



(b) Kinematic viscosity, 260 centistokes; initial liquid level, 4 inches.

Figure 9. - Concluded. Comparison of empirical correlation $\Delta h_w/\Delta H_w = 1 - e^{-m\frac{\nu t}{R^2}}$ and experimental data.